MECHANICAL DRAWING

S. A. MORTON

A
64
16
ct

QA
4

MOUNT ALLISON
UNIVERSITY
MEMORIAL LIBRARY

LITTERAE RELIGIO SCIENTIA

103048

BORDER MASTER CHARLES CHAMBERS KINGS COLLEGIATE WINDSOR N.S.

HIS FIRST TERM [illegible] LAST [illegible] SCHOOL [illegible]

QA
401
M2
T

MECHANICAL DRAWING

BY

S. A. MORTON, M.A.
TEACHER OF MATHEMATICS IN HALIFAX ACADEMY

◆

T. C. ALLEN & CO.
HALIFAX, N.S.
1905

QA
454
M6

103048
Mount Allison
Memorial
Library

PREFACE

MECHANICAL drawing, in addition to its value from a strictly practical point of view, is also of importance as affording an excellent introduction to the study of geometry. Through it the pupil discovers for himself many of the fundamental principles of geometry, so that later on, when he begins the theoretical study of this science, many of its difficulties have been removed.

Mechanical drawing is a subject at which all can do something. A pupil, dull it may be in other subjects, in this can attain a fair measure of success. He needs to show diligence, neatness, and accuracy in his work, and the acquiring of these habits is in itself of educational value.

The constructions in this book are of an elementary nature, and, for the most part, are confined to the subject-matter of the first book of Euclid. More complex problems have a place in books adapted to the needs of students in technical schools and colleges.

The aim of the author throughout has been to be practical. and this aim it is believed, has been realized, especially in Chapter XI. The exercises of this chapter have been done in a manual training school; and a boy who fully understands them and is of a mechanical turn can take the measurements here given, draw the required diagrams, and with his wood-working tools make the different objects.

Part I is intended for pupils of Grades VII and VIII of the schools of Nova Scotia, and Part II for those of Grades IX and X. Chapter XI is intended for the use of manual training pupils only.

The answers to the exercises have been got by calculation and may slightly differ from the answers obtained by plotting. In his measurements the pupil must be as exact as can reasonably be expected. When the value of a line is expressed in three figures, his answer should agree with the first two figures and be not more than one away in the last figure. For example, if he obtains 4.36 or 4.38 inches, and 4.37 inches is given as the answer, his work may be said to be correct. In measuring an angle he should get a result not more than one degree different from the answer. His value for an area, however, may not so closely approximate that got by calculation; for, as the area of a figure is the product of two numbers, a slight difference in the value of one of the numbers may make a considerable difference in the product.

CONTENTS

PART I

CHAPTER PAGE
I. INSTRUMENTS, LINES, ANGLES 1
II. PROBLEMS ON STRAIGHT LINES AND ANGLES . . 10
III. TRIANGLES 20
IV. QUADRILATERALS 28
V. AREAS 37
MISCELLANEOUS EXERCISES 42

PART II

VI. POLYGONS 48
VII. PRACTICAL APPLICATIONS 53
VIII. SCALES 59
IX. LOCATION OF POINTS 68
X. PROJECTION OF POINTS AND LINES 77
XI. WORKING DRAWINGS 90
MISCELLANEOUS EXERCISES 99
ANSWERS 107

PART I

CHAPTER I

INSTRUMENTS

1. Pencils. — Practical draughtsmen make their drawings in pencil and ink. They use hard pencils by which fine lines are drawn, and afterwards Indian ink to make them more distinct. This ink has advantages over ordinary ink in that it dries quickly and retains its color. It is not necessary for those for whom this book is intended to make their drawings in ink. They had better use pencils of medium hardness and be careful to keep them nicely pointed.

2. Ruler. — The ruler should be twelve inches long and have on it the inch and metric divisions, the diagonal scale in inches, half inches, and centimeters, and the scale of chords.

3. Dividers. — Dividers are used for measuring lines and, when one leg is furnished with a pencil point, for making circles. In taking measurements the pressure of the fingers is apt to cause the legs of the dividers to come too close together. This may be guarded against by having the distance between the points regulated by a screw.

4. Protractor. — A protractor for setting out and measuring angles is necessary. There are two kinds, the rectangular and the semicircular. The rectangular is engraved on some rulers. The semicircular is made of horn or metal. The horn protractor is preferred, and its use is recommended whenever the student is required to draw one line at right angles to another.

These few instruments are inexpensive, and by means of them all the constructions in this book may be drawn.

Paper. — White unruled paper will do for all purposes of this book. The drawing paper used by architects is expensive and beyond the means of most students. Paper of a much cheaper kind may be used. It is essential that it be clean and have its edges evenly cut.

Notes. — Neatness and accuracy must be insisted upon. It is highly important for the student to do his work *well*. If he fails in this at first, he must try again before proceeding farther. The student is to draw for himself the figures belonging to the problems according to the directions given, and also those of the various exercises. In making the diagrams, draw given lines and lines of construction thin; and those required, thick. Parts of lines not needed in the finished drawing or to show the method of construction should be carefully erased with rubber. The value of a line, or an angle, or whatever is required, should closely agree with that obtained by a mathematical calculation. The numerical values of all parts given and of all parts required should be neatly entered on the diagrams.

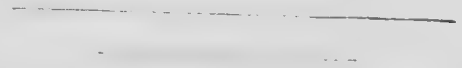
LINES

In drawing straight lines from one point to another great care should be taken that the line begins *exactly* at one point and ends *exactly* at the other; and that it is *straight* and of uniform width throughout its whole length. To secure these ends the pencil should be held nearly straight up and down, and should closely follow the direction of the ruler, and the pressure upon it should be the same from one point to the other. The line should be drawn from the left to the right and with the aid of the ruler. Before drawing the line see that the ruler is so placed that the pencil point will pass exactly through the points. Do not measure a line by means of your ruler only. Use your dividers as well. Be sure that one point of the dividers is exactly at one end of the line and that the other point is exactly at the other end; and that, while the length of the line is being transferred to the scale, the ends of the dividers do not come closer together. Be careful also not to press so hard upon the dividers as to puncture the paper.

EXERCISE I

(NOTE. — Answers are to be correct to the second place of decimals.)

1. Measure and draw these lines:



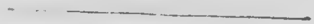
2. Draw a line equal to the sum of the lines in No. 1, and measure it.

3. Draw a line equal to their difference and measure it.

4. Draw three straight lines whose lengths are respectively 2.37, 1.68, and 3.94 inches.

5. Draw three lines of different lengths. First estimate their lengths and then measure them. How near were the estimated lengths to the exact lengths?

6. Draw lines whose lengths are respectively $2\frac{7}{12}$, $1\frac{1}{3}$, and $\frac{7}{8}$ inches.

7. Estimate the length of this line: 
How near does your estimate come to the exact length? Draw a line three times as long and measure it.

8. Estimate the length of this page. Draw a line as long, measure it, and record your answer in (1) inches and decimal of an inch, and (2) decimeters and centimeters.

9. Draw a line whose length you judge to be 2¼ inches.

10. Draw a line whose length you judge to be 8 centimeters.

ANGLES

AB and *AC* are straight lines drawn from the same point *A*. These two lines form an *angle* at *A*. The angle is named *BAC*, or *CAB*. *AB* and *AC* are the *arms* of the angle *BAC*.

The size of an angle depends upon the opening between the arms and not upon their lengths.

Angles are plotted and measured usually by means of the protractor. The scale of chords is sometimes used. Unless otherwise directed the pupil is to use the protractor.

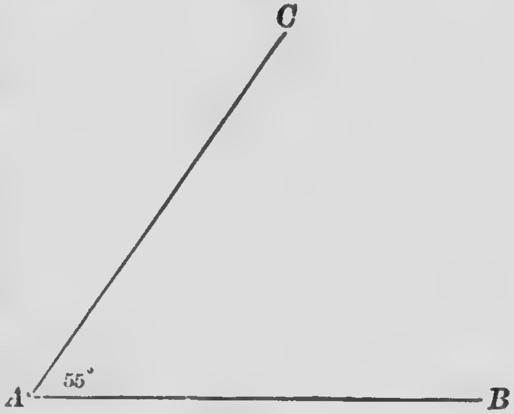


FIG. 1.

If you examine a semicircular protractor, you will see that its circumference is divided into 180 equal divisions numbered at intervals of 10 from 0 to 180. These divisions are called **degrees**, of which the sign is °. For instance, 40° is read "40 degrees." You will also notice that the middle point of the straight edge of the protractor is marked. For the sake of reference call this point *O*.

If it be required to make any angle — for example, 55° — proceed thus: Take a line *AB* (Fig. 1) and on it place the protractor so that *O* is at *A* and the straight edge to the right of *O* is on the line *AB*. Then make a small pencil mark on the paper at the division on the circumference whose number is 55. Draw a line from this mark to *A*. Then at *A* there is an angle of 55°.

Conversely, if it be required to measure any angle — for example, the angle *BAC* of Fig. 1 — proceed thus: As before, place *O* on *A* and the straight edge on *AB*. Then notice where *AC* meets the circular edge. It is at the division 55. Then *BAC* is an angle of 55°. If the line *AC* is so short that it does not meet the circular edge, with the pencil extend *AC* until it does meet the edge, and then read the number of degrees in the angle.

The protractor is so numbered and divided that angles of any size between 0° and 180° can readily be plotted or measured. When the size of an angle is 90°, the angle is a *right angle*.

EXERCISE II

1. Plot angles of ~~35°, 80°, 110°, 130°, and~~ 160°.

2. On your paper take a point and from it draw a line twice as long as *AB* and a line twice as long as *AC*, making the angle between them twice as large as the angle in Fig. 1.

3. Name the angle in No. 2. *DEF*. Draw a line from *D* to *F*. Measure it and the angles at *D* and *F*.

4. What is the sum of the values of the angles at *E*, *D*, and *F*? Your answer should be 180°.

5. Draw five angles of different sizes. First estimate their sizes and then measure them. How near were your estimates?

6. Take a point on your paper and from it draw a line directly upwards. You have drawn a *vertical* line.

7. From the point in No. 6 draw, also, a line directly towards the right. You have now drawn a *horizontal* line. What is the angle between these two lines?

8. Take a point *A* and from it draw a line directly upwards and 1½ inches long; a line directly towards the right and 2 inches long; and a line directly towards the left and 1 inch long. How many angles are formed at *A*? Measure each. What is their sum?

9. Measure each of the angles in Fig. 2. What is their sum?

10. Make a drawing of your own suggested by Fig. 2. Measure each of the angles. What is their sum?

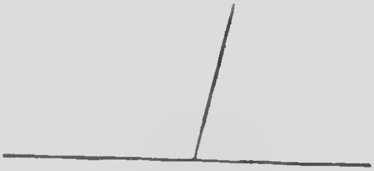
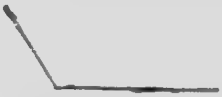
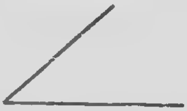


FIG. 2.

EXERCISE III

1. Measure these angles and find an angle equal to their sum, and an angle equal to their difference.





2. Make an angle three times the size of that in Fig. 1.

3. On your sheet of paper describe angles of 32°, 108°, 140°, 45°, 67°, and 23°.

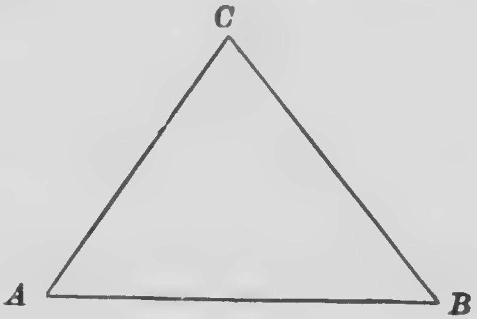


Fig. 3.

4. Figure 3 is that of a *triangle*. Measure the angles at *A*, *B*, and *C*. How many degrees are there in the three angles all together?

5. Make a triangle of your own and letter it as in Fig. 3. How large is each angle and what is the sum of all?

6. Keep your triangle of No. 5. Extend the line *AB* 1 inch towards the right, and call the end of the extended part *D*. Measure the angle *CBD*. Also find an angle equal in size to *A* and *C* together. Compare the two values.

7. Take a straight line *AB*, 3.6 inches long. Cut off a part *AC*, 1.8 inches long. Through *C* draw *CD*, 2.8 inches long, making the angle *BCD*, 72°. Draw lines from *D* to *A* and *B*. Measure their lengths and the angles of the triangle *ADB*.

8. Look closely at the dial of a clock. What is the angle between the hands at one o'clock? At three o'clock? At half-past six? At eight o'clock? At ten o'clock? Draw each of these angles on your paper.

9. On a map are three places which we will call *A*, *B*, and *C*. *B* is 2 inches due north of *A*, and *C* is $2\frac{7}{8}$ inches due west of *A*. Represent on your paper the positions of these places. How far is *B* from *C*?

10. Draw a horizontal line 2 inches long. At one end

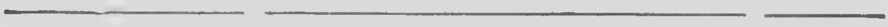
erect a vertical line 1.45 inches long, and at the other end a vertical line 2.36 inches long. Join the upper end of the one vertical line to that of the other. Measure each of the angles last made. What is their sum?

EXERCISE IV

1. On the scale of 1 inch = $\frac{1}{4}$ inch draw a horizontal line 1 foot 3 inches long and a vertical line 9 inches long.

2. Use your diagonal scale and draw lines to represent these distances: 463, 120, 49, 376, 87, and 245 feet.

3. If 100 feet be represented by a line 1 inch in length, what does each of the following lines represent?



4. A man started from a certain point and walked directly towards the east 204 feet. Then he turned and walked 167 feet directly towards the north. How far is he now from the starting-point? Draw a diagram to represent this.

5. Measure the top of your desk and, on a reduced scale, represent it on your paper.

6. A man walks 2 miles; then he turns to his right hand through an angle of 90° and walks 3 miles; and then he turns to his left through an angle of 90° and walks 4 miles. Draw the diagram, on the scale of a mile to an inch. How far is he at the end of the time from his starting-point?

7. A wall is 16 feet high. The foot of a ladder is 12 feet out from the wall and the top of the ladder just reaches the top of the wall. Represent this. How long is the ladder?

//

8. Two straight roads branch off making an angle of 40° with each other. A house is on the one road 6 miles from the corner, and a house is on the other road 8 miles from the corner. How far apart are the houses? Represent this on the scale of 2 miles to 1 inch.

9. A flag-staff is 48 feet high. Two guy-ropes are fastened to the top and carried to the ground to points 12 feet and 18 feet respectively from the foot of the staff, and directly opposite to one another. Represent this on a scale of 12 feet = 1 inch. Find the length of each rope and the angle each makes with the ground.

10. Four roads start at the same point. The first runs due north, the second due east, the third due south, and the fourth due west. Locate a point 4 miles along the first road, a point 5 miles along the second, a point [illegible] miles along the third, and a point 8 miles along the fourth. Find the distances of the first and third points from the second and fourth. Use any convenient scale.

CHAPTER II

PROBLEMS ON STRAIGHT LINES AND ANGLES

Problem 1. To bisect this line

that is, to divide it into two equal parts.

There are several methods of doing this.

1. *By Measurement.* Find the exact length of the line. It is 2.08 inches. Half of the line will be 1.04 inches long. From either end of the line measure off a distance of 1.04 inches. You have then got its middle point. This is a good exercise to test accuracy of measuring.

2. *By Trial.* Estimate the length of half the line. With the dividers see if you are right. If not, try again. By repeated trial you can find its middle point. After a little practice the eye will be so trained in estimating distances that only a few trials will be necessary.

3. *By Means of Protractor and Dividers* (Fig. 4). At A, make an angle of any size. Let us say 34°. At B, make an angle of the same size. Make AC of any length, and $BD = AC$. Draw a line from C to D. Then E is the middle point of AB.

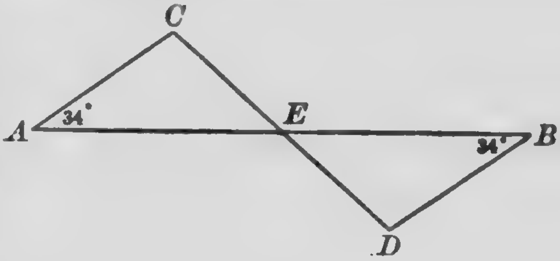


FIG. 4.

4. *By Means of Compasses* (Fig. 5). Extend the compasses until the distance between the points is greater than half the line. Place one point of the compasses at *A*, and with the other point trace part of the circumference of a circle; that is, trace an *arc* of a circle. Be careful that the stationary point of the compasses does not puncture the paper. Again, place one point at *B* and, with the compasses as far apart as before, that is, with the same *radius* as before, describe an arc. These arcs cut one another at *C* and *D*. Draw a line from *C* to *D*. Then *E* is the middle point of *AB*.

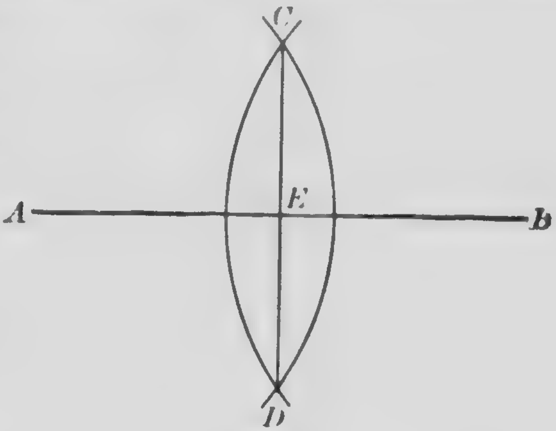


FIG. 5

NOTE. — You are to bisect this line in the four ways. After this when you are required to bisect a line, if no method is given, you may use any one you wish.

EXERCISE V

1. In Fig. 4, show by measurement, that $CE = DE$, angle at $C =$ angle at D, and angle $CEA =$ angle BED.

2. In Fig. 5, show in a similar way, that $CE = DE$, and that each angle at $E = 90°$.

3. In Fig. 5, join C and D to A and B, and show that the four lines just drawn are equal to one another.

4. Measure the top of your desk, and stick pins in the middle points of the sides.

5. Using a scale of your own, draw a diagram to represent No. 4.

6. Take a line as long as this

and divide it into quarters by bisecting it, and then again bisecting each half. Do this (1) by trial, (2) by measurement.

7. A wire is straight for $1\frac{1}{2}$ inches, then goes at an angle of 90° for 4 inches, then goes straight to the middle point between where it started and where it first turned off. How long is the wire? Draw the diagram.

8. Place these lines

so that they bisect each other and form angles of 90° each.

9. Take a line, AB, 3 inches long, and bisect it in C; from C draw CD, 2.4 inches long, to make with BC an angle of 58°. Join AD and DB. Measure their lengths and the angles at A and B.

10. AB is 2.34 inches, and AC is 3.02 inches, and the angle at A is 64°. D is the middle point of AB, and E is the middle point of AC. How far are B from C and D from E? Compare your two answers.

PROBLEM 2. To draw a straight line at right angles to AB from C, a point in AB (Fig. 6).

1. *With the Protractor.* At C, make ACF an angle of 90°. Then CF is at right angles, or *perpendicular*, to AB.

2. *With the Compasses.* With C as centre, and with a suitable radius, draw arcs to cut AB in D and E. With D and E as centres, and with a radius larger than before, describe arcs cutting each other at F. Join F to C. Then CF is at right angles to AB.

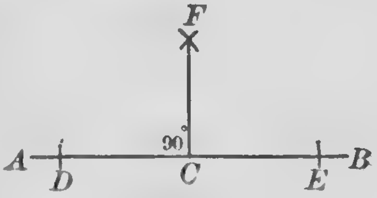


FIG. 6.

PROBLEM 3. To draw a straight line at right angles to AB from C, a point not in AB (Figs. 7, 8).

1. *With the Protractor* (Fig. 7). Take any point D in AB. Make ADE an angle of 90°. Join D to C. Measure the angle EDC, and make the angle DCF as large. Then CF is perpendicular to AB.

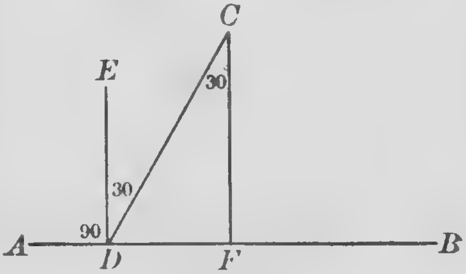


FIG. 7.

2. *With the Compasses* (Fig. 8). With C as centre, and with any suitable radius, cut AB in D and E. From these points as centres, and with a suitable radius, describe arcs cutting at H. Join C to H and extend CH till it meets AB in F. Then CF is perpendicular to AB.

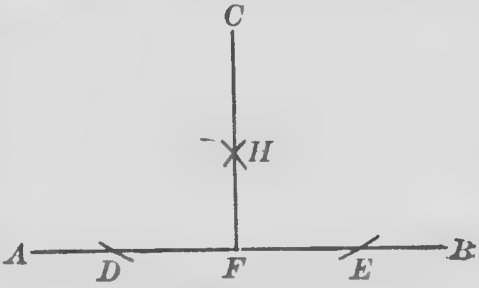


FIG. 8.

EXERCISE VI

1. Enlarge Fig. 6 to about twice its size.

2. Join *F* to *D* and *E*. Measure the lengths of *FD* and *FE* and the angles of the triangles *FCD* and *FCE*.

3. Bisect *FD* in *G* and join *C* to *G*. Show that $CG = GD = GF$, and that the angle $DGC = 90°$.

4. Bisect *FE* and proceed in a manner similar to that in No. 3.

5. If a man walks 5 miles in one direction, and then turns through an angle of 90° and walks 12 miles in that direction, how far is he from his starting-place? Show by a diagram.

6. Make a triangle somewhat like that of Fig. 3. Bisect each side, and at the middle point of each side make a right angle. If your work is correct the lines drawn at right angles to the sides will pass through the same point.

7. Draw a line 3 inches long, assume a point about $1\frac{1}{2}$ inches above it, and from this point draw a perpendicular to it.

8. Make a triangle somewhat like that of Fig. 3. Draw a line from *C* perpendicular to *AB*, one from *A* perpendicular to *BC*, and one from *B* perpendicular to *AC*. If your work is right, the perpendiculars will pass through the same point.

9. Take a line 2 inches long and at one end erect a perpendicular 2 inches long. Join the ends of the lines and show that the triangle formed has two angles of 45° each.

10. Draw a horizontal line 1.51 inches long. At the

ends erect perpendiculars whose lengths are 1.43 and 1.02 inches respectively. Produce the lines joining the ends of the perpendiculars until they meet. How far is the point where they meet from the ends of the perpendiculars?

PROBLEM 4. Through *C* to draw a line parallel to *AB* (Fig. 9).

Join *C* to any point *D* in *AB*. Measure the angle *CDB* and make the angle *DCE* equal to it. Then *EF* is parallel to *AB*.

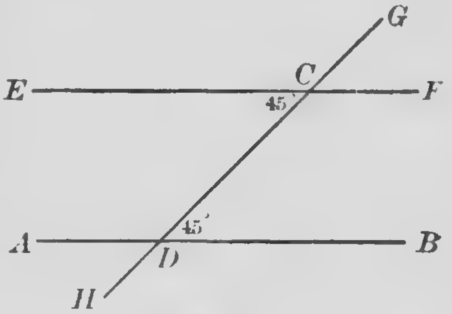


FIG. 9.

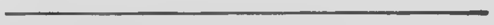
NOTE.—*AB* and *EF*, if produced ever so far in either direction, do not meet. They are *parallel* lines.

PROBLEM 5. To draw a line parallel to *AB* and .623 inch from it (Fig. 10).

FIG. 10.

At *A* and *B* make angles of 90°. Make *AC* and *BD* .623 inch long each. Join *C* to *D*. Then *CD* is parallel to *AB*.

PROBLEM 6. To divide the line



into five equal parts.

1. *By Measurement.*

2. *By Trial.* These methods are similar to those of Problem 1.

3. *By Means of Dividers and Protractor* (Fig. 11). Measure the given line and make *AB* as long. Draw *AC* at any convenient angle and length, and from *A* set off five equal distances. Join 5 to *B*. Measure the angle at 5, and at the other points of division make angles equal to it. Then *AB* is divided into five equal parts.

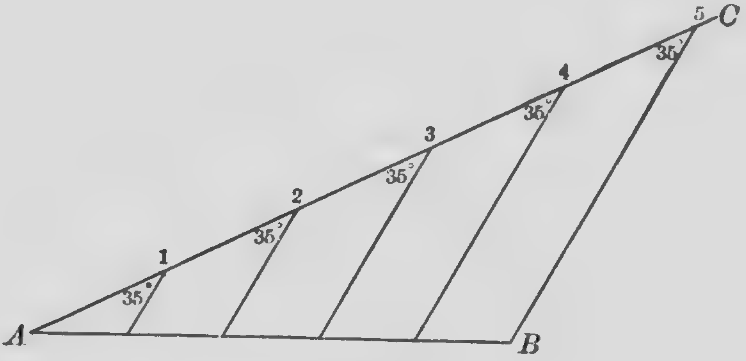


FIG. 11.

EXERCISE VII

1. Make a drawing like Fig. 9. Measure all the angles in it. What angles are equal to one another? What pairs of angles are equal to 180°?

2. Are the edges of your ruler and sheet of paper parallel? Prove your answers.

3. Draw three parallel lines $\frac{3}{4}$ inch apart.

4. Draw a line $3\frac{1}{4}$ inches long, and from it cut off $\frac{1}{5}$ of its length. Do this by means of dividers and protractor.

5. Draw a line $2\frac{3}{4}$ inches long, and through a point *A*, 1 inch from it, draw another line $2\frac{1}{2}$ inches long, parallel to it and bisected at *A*.

6. Draw any triangle you wish. Bisect two sides and join their middle points. The line now drawn is parallel to the third side and equal to half of it. Prove.

7. Make a drawing somewhat like Fig. 10. Take *AB* $2\frac{2}{5}$ inches long, and *AC* 1 inch long. Join *BC* and *AD* and find their lengths.

8. In a drawing like Fig. 9, if the angle *CDB* is 48°, and *CD* is $1\frac{3}{4}$ inches long, how far apart are the lines?

9. One road is parallel to another. A straight path runs from one to the other, is 1200 feet long, and makes an angle of 110° with the roads. What is the distance between the roads?

10. Draw two parallel lines $2\frac{1}{2}$ inches apart, and between them place two lines so that there shall be four equidistant parallel lines.

PROBLEM 7. To bisect the angle *BAC* (Fig. 12).

1. *By Means of the Protractor.* Measure the angle *BAC* and make the angle *CAD* half as large. Then *AD* bisects the angle *BAC*.

2. *By Means of the Dividers.* With *A* as centre and with any convenient radius draw the arc *BC*. With *B* and *C* as centres and with any radius draw arcs to cut one another at *D*. Join *D* to *A*. Then *AD* bisects the angle *BAC*.

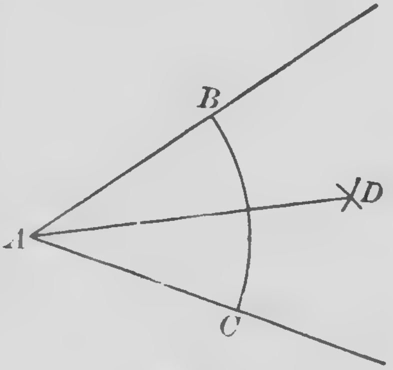


FIG. 12.

PROBLEM 8. To trisect the right angle *BAC* (Fig. 13).

1. *By Means of the Protractor.* A right angle equals 90°. Make each of the angles *HAD* and *DAE* equal to 30°. Then the right angle is trisected.

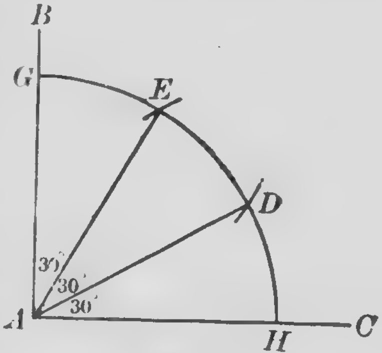


FIG. 13.

2. *By Means of the Dividers.* With *A* as centre and with any convenient radius draw the arc *GH*. With the same radius, and with *G* and *H* as centres, draw arcs cutting *GH* in *D* and *E*. Join *A* to *E* and *D*. Then the right angle *BAC* is trisected.

EXERCISE VIII

1. Draw a straight line *AB*, 2.68 inches long, and from a point *D*, .7 inch from *A*, draw a line *DC*, making an angle of 54° with *DB*, and bisect the angle *CDB*.

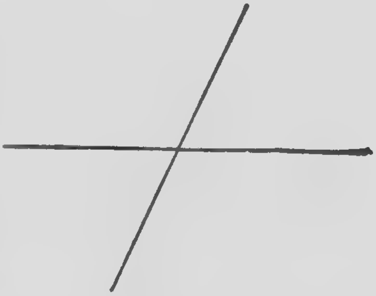
2. Set off with the protractor an angle of 63°. Bisect this angle and again bisect each half.

3. Draw a triangle. Bisect all three angles. The bisecting lines will be found to go through the same point.

4. From the point found in No. 3 draw perpendiculars to the sides. If your work is correct, these perpendiculars are equal to one another.

5. Make an angle of 72° and bisect it. Take a point on the bisector 2½ inches from the angle and measure the lengths of the perpendiculars drawn from the point to the arms of the angle.

6. Draw an angle of 114° and trisect it.



7. Measure the angles in this figure. Bisect each angle. Measure the angles formed by the bisectors. What do you notice about them?

8. Draw a vertical and a horizontal line, each 1.86 inches long, to bisect one another at *A*. Bisect each of the four angles formed. Your figure will represent a compass with eight points. Put letters at the ends of the lines, beginning with the upper and going round by the right. Let these letters be *N*, *NE*, *E*, *SE*, *S*, etc.

9. A vessel left a port and sailed 10 miles directly south. Then it turned and sailed 15 miles directly east. Then it sailed 5 miles southeast. Then it sailed 20 miles northeast. How far is it now from the port left? Draw a diagram to show the course the vessel took.

10. A road runs from *A* to *B* east 6 miles, then goes to the northwest from *B* to *C* 8 miles, and then goes to the north from *C* to *D* 12 miles. Draw a diagram to show the road, and measure the direct distance from *D* to *A*.

CHAPTER III

TRIANGLES

A TRIANGLE is shown in Fig. 3. When the three sides are equal, the triangle is *equilateral.* The three angles are also equal. You have seen that the sum of the three angles of a triangle is 180°. Therefore each angle of an equilateral triangle is 60°.

When only two sides of a triangle are equal, the triangle is *isosceles.*

PROBLEM 9. To make an equilateral triangle on a base 1.21 inches long (Fig. 14).

Take a line, 1.21 inches long, and proceed in one of these ways: (1) *With the dividers.* Spread the points of the dividers 1.21 inches apart, and with A as centre, describe an arc; and with B as centre, describe an arc. These arcs intersect at C. Join C to A and B. Then CAB is an equilateral triangle. (2) *With the protractor.* At A and B make angles of 60°. Produce the arms of the angles to meet at C. Then ABC is an equilateral triangle.

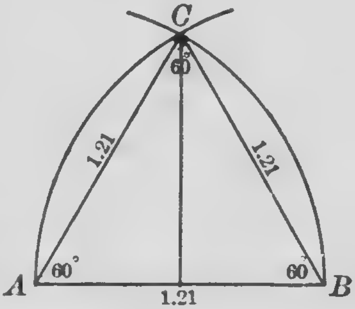


FIG. 14.

PROBLEM 10. To make an isosceles triangle on a base 1.86 inches long and with a side 1.28 inches long (Fig. 15).

Take *AB*, a line 1.86 inches long. Spread the points of the dividers 1.28 inches apart, and with *A* as centre, describe an arc; and with *B* as centre, describe an arc. These arcs intersect at *C*. Join *C* to *A* and *B*. Then *CAB* is an isosceles triangle.

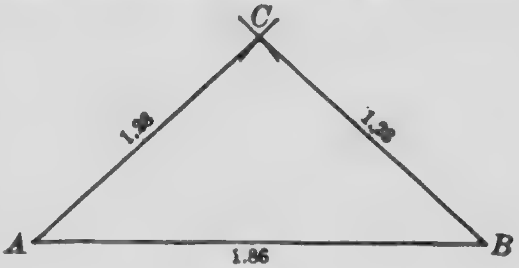


FIG. 15.

PROBLEM 11. To make a triangle whose sides shall be 1.27, 1.2, and 1.31 inches long, respectively (Fig. 16).

Take *AB*, a line 1.27 inches long. Spread the points of the dividers 1.2 inches apart, and with *A* as centre, describe an arc. Again, spread the compasses 1.31 inches apart, and with *B* as centre, describe an arc. These arcs intersect at *C*. Join *C* to *A* and *B*. Then *CAB* is the triangle required.

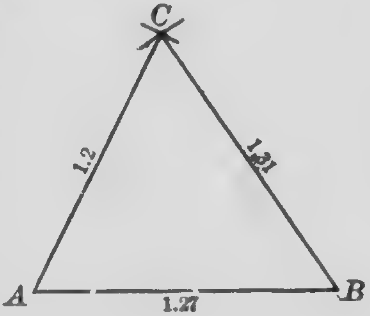


FIG. 16.

PROBLEM 12. To make a triangle when two sides are 1.56 and 1.32 inches long, respectively, and the angle between them is 65° (Fig. 17).

Take *AB*, a line 1.56 inches long. At *A* make an angle of 65°. Measure off *AC* 1.32 inches in length. Join *C* to *B*. Then *CAB* is the triangle required.

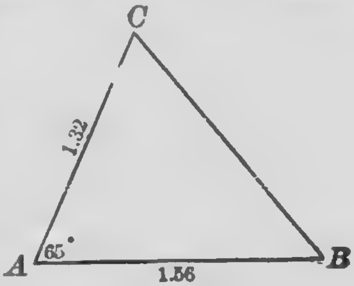


FIG. 17.

EXERCISE IX

1. How long is this line?

Draw a line as long, and upon it describe an equilatera triangle.

2. In Fig. 15, extend AC to any point D. Bisect the angle BCD and the angle ACB. Prove that the former bisector is parallel to AB, and that the latter makes an angle of 90° with AB.

3. How long are these lines?

Take a line as long as the former, and upon it describe an isosceles triangle having a side as long as the latter line.

4. Measure these lines:

and make a triangle with sides equal to them.

5. Enlarge each side of the triangle in Fig. 16 to twice its length, and measure the angles. Which is the largest? Which, the smallest?

6. How long are these lines?

Make a triangle with two sides as long as these lines, and with the angle between them equal to the smallest angle of the triangle in No. 5.

7. On a scale of your own, make a triangle whose sides shall be 48, 54, and 60 feet, respectively. Measure the angles. Which is the largest? Which, the smallest?

8. Make a triangle with sides of 3, 4, and 5 inches, and measure the angles. One of them is 90°. Which? Notice that $(3)^2 + (4)^2 = (5)^2$.

9. On a reduced scale, make a triangle with sides of 5,

12, and 13 inches, and measure the angles. One of them is 90°. Notice that $(5)^2 + (12)^2 = (13)^2$.

10. Make a diagram to represent a triangle with two sides 316 and 409 feet long, and the angle between them 137°. Also, measure the third side and the other angles.

PROBLEM 13. To make a triangle on a base 1.71 inches long, having the two angles at the base 67° and 42°, respectively (Fig. 18).

Take a line 1.71 inches long, and at its ends draw lines, making angles of 67° and 42°. Produce the lines to meet. Then the required triangle is drawn.

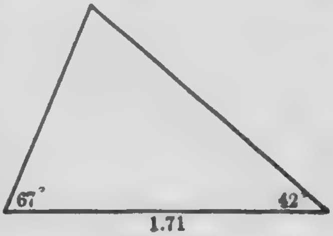


FIG. 18.

PROBLEM 14. To make a triangle on a base 1.71 inches long, having one angle at the base 67°, and the vertical angle 71°.

The three angles of a triangle are equal to 180°. In this problem the other angle at the base = 180° − (67° + 71°) = 42°. Then the problem is similar to Problem 13.

PROBLEM 15. To make a triangle on a base of 1.73 inches, having one angle at the base 61°, and having a height of 1.29 inches.

Take *AB*, 1.73 inches in length. Draw *AD* 1.29 inches long, to make an angle of 90° with *AB*. Make *ADC* an angle of 90°, and *BAC* an angle of 61°. Then *CAB* is the triangle required.

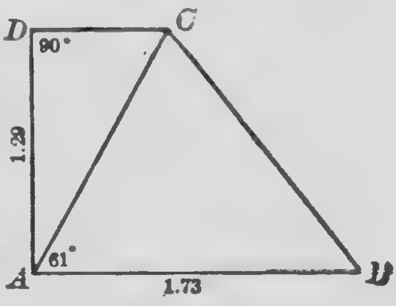


FIG. 19.

Problem 16. To make a triangle, having given two sides 1.83 and 2.45 inches long, and the angle opposite to the smaller side 34° (Fig. 20).

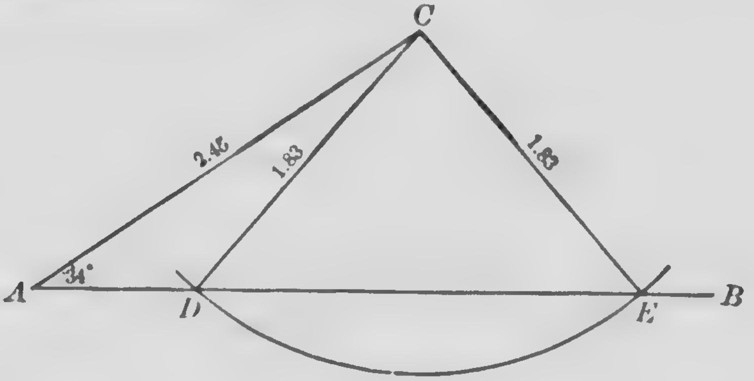


Fig. 20.

Take *AB*, a line of an indefinite length. Make *BAC* an angle of 34°, and *AC* 2.45 inches long. With *C* as centre and with the points of the dividers 1.83 inches apart, describe an arc cutting *AB* in *D* and *E*. Join *C* to *D* and *E*. Then, either the triangle *ACD* or the triangle *ACE* will suit the values given.

Note. — When the given angle is opposite to the smaller of the two sides, two solutions are possible: except when the arc does not cut *AB* in two points but merely touches it. Then the angle at *D* becomes 90°.

Problem 17. To make an isosceles triangle on a base of 1.8 inches, and with a height of 1.24 inches (Fig. 21).

Take a line 1.8 inches long, bisect it, and erect a perpendicular 1.24 inches long. Then join the points as in the figure.

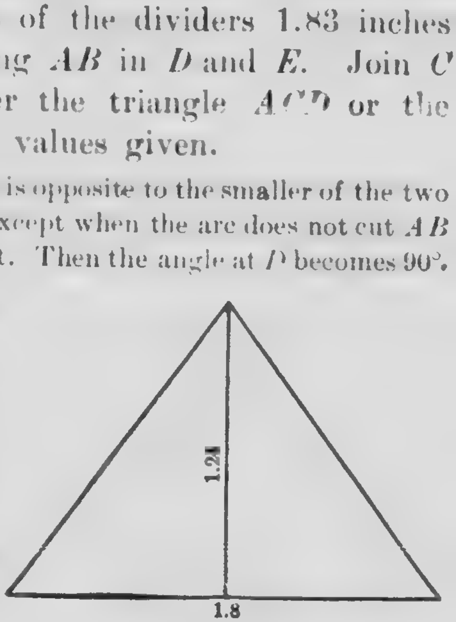


Fig. 21.

Problem 18. To make an equilateral triangle with a height of 1.31 inches (Fig. 22).

Take a line *CD*, 1.31 inches long. Through *D* draw *AB* to make an angle of 90° with *CD*. At *C* make angles of 30° on either side of *CD* as in the figure. Then *CEF* is the equilateral triangle required.

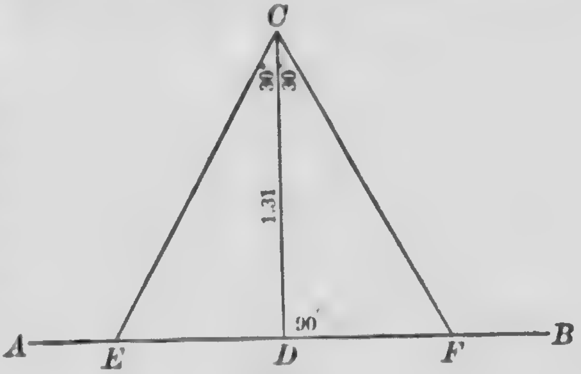


Fig. 22.

Problem 19. To make an isosceles triangle with a height of .96 inch and a vertical angle of 86° (Fig. 23).

This is similar to Problem 18. Its construction is easily seen from the figure.

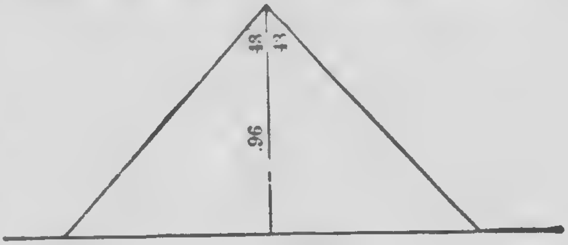
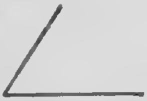
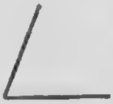


Fig. 23.

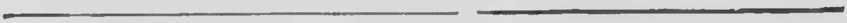
EXERCISE X

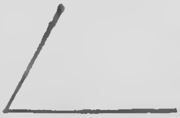
1. How long is this line? How large are these angles? Make a triangle whose base is as long as this line, and whose angles at the base are as large as these angles; and measure the other sides and angle.





2. How long are these lines?

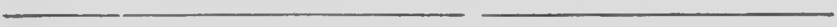




How large is this angle? Make a triangle whose base is equal to the first line, whose height is equal to the second line, and one of whose angles at the base is as large as this angle. Also, measure the other sides and angles.

3. Make two triangles two of whose sides are equal to the lines in No. 2, the angle opposite the smaller line being equal to that in No. 2.

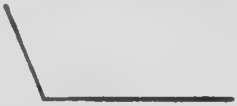
4. How long are these lines?

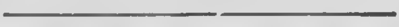


Make an isosceles triangle, whose base is equal in length to the first, and whose height is equal to the other line. Also, measure the other sides and the three angles.

5. Make an equilateral triangle whose height is equal to twice that of the isosceles triangle in No. 4.

6. How large is this angle?



How long is this line?  Make an isosceles triangle whose height is as long as this line and whose vertical angle is as large as this angle.

7. Describe a triangle having two sides $2\frac{1}{2}$ and $3\frac{3}{4}$ inches long, respectively, the angle opposite the second being 85°.

8. Draw a triangle, the sides of which are 2.5, 3, and 3.5 inches, and make another triangle equiangular to it on a base of 3.75 inches.

9. Draw an isosceles triangle having the angles at the base each 72°, and the two equal sides each $2\frac{1}{2}$ inches long. Measure the base.

10. Bisect a straight line *AB*, $3\frac{1}{4}$ inches long, in *C*; through *C* draw *CD*, 2.4 inches long, making the angle *DCB*, 63°. Join *D* to *A* and *B*. Measure *DA* and *DB*, and the angles of the triangle *ADB*.

CHAPTER IV

QUADRILATERALS

Note. — *Quadrilaterals* are figures bounded by four straight lines.

Problem 20. To make a square on a line 2.16 inches long (Fig. 24).

Take *AB*, a line 2.16 inches long. At each end draw a line upwards to make an angle of 90° with *AB*. Make the lines *AD* and *BC* each 2.16 inches long. Draw a line from *C* to *D*. Then *ABCD* is the square required.

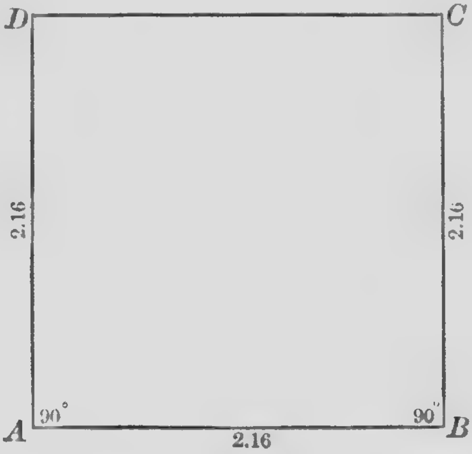


Fig. 24.

Note. — A *square* has all its sides equal and all its angles 90° in size. The *diagonals* are the lines joining opposite corners.

PROBLEM 21. To make a rhombus on a base 1.67 inches long, and with one of the angles at the base 69° (Fig. 25).

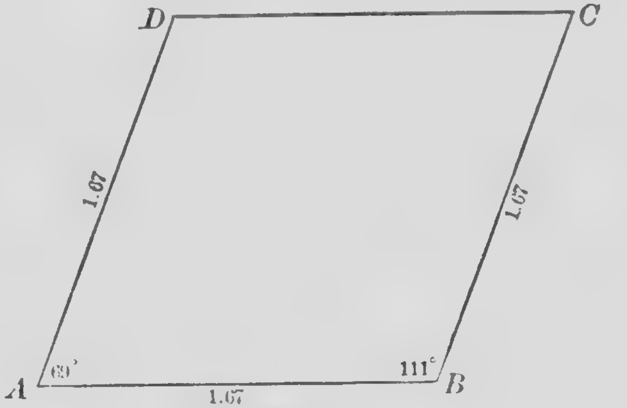


FIG. 25.

Take *AB*, a line 1.67 inches long. At *A* make the angle *BAD* 69°, and at *B* make the angle *ABC* 111°. Make *AD* and *BC* each 1.67 inches long. Draw a line from *C* to *D*. Then *ABCD* is the rhombus required.

NOTE. — A *rhombus* has all its sides equal, but no angle of 90°. If one angle is 69°, the angle near it is 180° − 69°; that is, is 111°.

PROBLEM 22. To make a rectangle whose adjacent sides are 2.25 and 1.53 inches long respectively (Fig. 26).

The method of drawing is easily seen from the accompanying figure.

NOTE. — A *rectangle* has its opposite sides parallel and all its angles 90° in size.

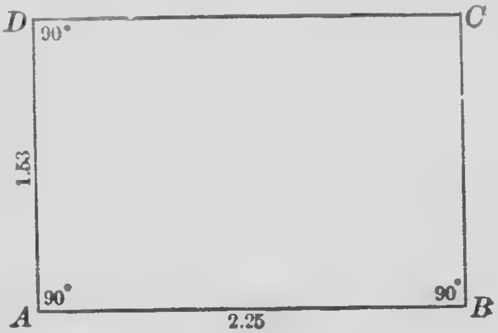


FIG. 26.

PROBLEM 23. To make a parallelogram whose ad cent sides are 2 and 1.18 inches long respectively a meet at an angle of 73° (Fig. 27).

Take *AB*, a line 2 inches long. At *A* make the angle *BAD* 73°, and at *B* make the angle *ABC* 107°. Make *AD* and *BC* each 1.18 inches long. Join *C* to *D*. Then *ABCD* is the parallelogram required.

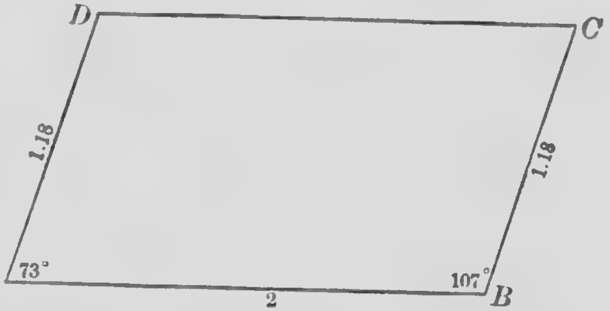


FIG. 27.

NOTE. — A *parallelogram* has its opposite sides parallel. If one angle is 73°, an adjacent angle is 107°. 107 = 180 − 73.

When one angle of a parallelogram is 90°, the figure is a *rectangle*.

When two adjacent sides of a rectangle are equal, the figure is a *square*.

When two adjacent sides of a parallelogram are equal and meet at an angle other than 90°, the figure is a *rhombus*.

EXERCISE XI

1. In Fig. 24, if *AC* and *BD* be drawn, show that they are equal and bisect one another at angles of 90°.
2. In Fig. 25, if *AC* and *BD* be drawn, show that they are not equal, but bisect one another at angles of 90°.
3. In Fig. 26, if *AC* and *BD* be drawn, show that they are equal and bisect one another.
4. In Fig. 27, if *AC* and *BD* be drawn, show that they are not equal, but bisect one another.
5. In Fig. 24, bisect *AB*, *BC*, *CD*, and *DA* in *E*, *F*, *G*, and *H*, respectively. Show that the figure *EFGH* is a square.

6. Similarly in Fig. 25, show that the figure *EFGH* is a rectangle.

7. Similarly in Figs. 26 and 27, show that the figure *EFGH* in each is a parallelogram.

8. Lettering a parallelogram as in Fig. 27, take *AB* 3.46 inches long, *AD* 2.05 inches long, and angle at *A* 57°. Bisect *AB* in *E* and *CD* in *F*. Join *AC*, *ED*, and *BF*, and measure the segments into which *AC* is divided.

9. Take a line 5.6 centimeters long, and on it describe a square.

10. Make a rhombus on a base of 2.74 inches and with one of the angles at the base 120°. Letter it as in Fig. 25. On *AB*, as a base, describe a triangle *ABE* so that *AE* shall equal 1.68 inches and *BE* 2.09 inches. Join *E* to *C* and *D*, and measure the lines *EC* and *ED* and the angles at *E*.

PROBLEM 24. To make a square, one of whose diagonals is 2.62 inches long.

From Example 1 of Exercise XI it is seen that the diagonals of a square are equal and bisect one another at angles of 90°. Take *AB*, 2.62 inches long, and bisect it in *E*. At *E* make *AEC* and *AED* angles of 90° each. Make *EC* and *ED* each 1.31 inches long. Join *A* and *B* to *C* and *D*. Then *ADBC* is the square required.

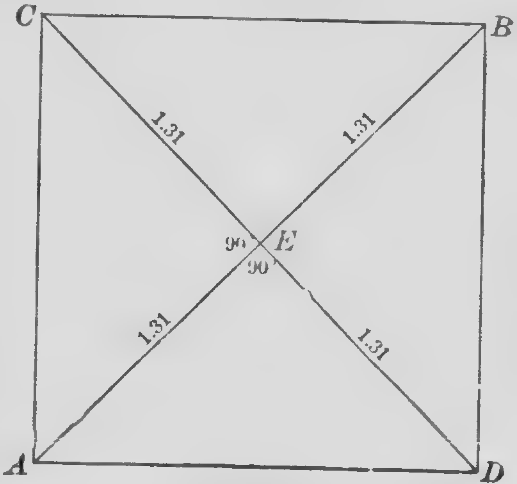


FIG. 28.

PROBLEM 25. To make a rhombus whose diagonals are 3.02 and 2 inches long respectively.

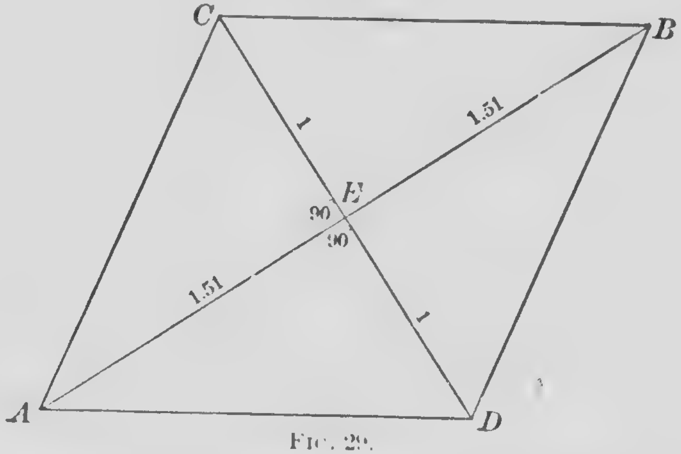


FIG. 29.

The diagonals of a rhombus bisect one another at angles of 90°. Take *AB*, a line 3.02 inches long, and bisect it in *E*. At *E* make *AEC* and *AED* angles of 90° each. Make *EC* and *ED* each 1 inch long. Join *A* and *B* to *C* and *D*. Then *ADBC* is the rhombus required.

PROBLEM 26. To make a rectangle whose diagonals are 2.82 inches each in length and inclined 49° and 131° to one another.

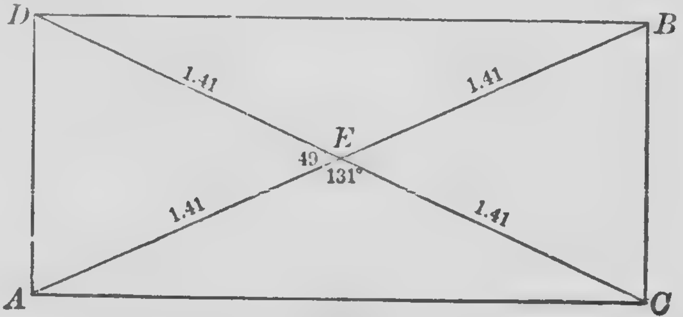


FIG. 30.

Take AB, 2.82 inches long, and bisect it in E. At E make AEC and AED angles of 131° and 49° respectively. Make EC and ED each 1.41 inches long. Join A and B to C and D. Then $ADBC$ is the rectangle required.

PROBLEM 27. To make a parallelogram whose diagonals are 3.44 and 1.72 inches long, respectively, and inclined at angles of 116° and 64° to one another (Fig. 31).

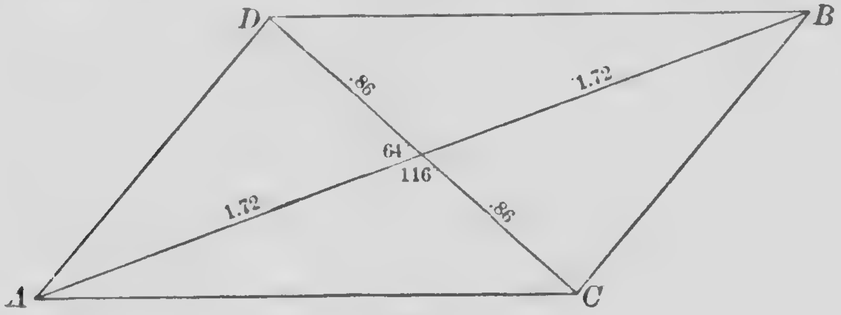


FIG. 31.

The method of drawing is easily seen from the accompanying figure.

EXERCISE XII

1. In Fig. 28, how long is AC or CB or BD or DA? How large is the angle CAD? CAB

2. In Fig. 29, how long is AC? How large is the angle CAD? How large is the angle ADB? ADC?

3. In Fig. 30, how long is AC? How long is AD? How large is the angle DAC? CAB?

4. In Fig. 31, how long is AC? How long is AD? How large is the angle DAC? How large is the angle ACB?

5. On a reduced scale, represent a diamond-shaped pane of glass whose greatest length is 12 inches and greatest breadth 5 inches.

6. Draw a parallelogram of which one side shall measure $2\frac{1}{2}$ inches, and the distance between it and the opposite side shall be $1\frac{1}{2}$ inches. One angle of the figure is to be 30°.

7. Take a line 1 inch long and on it draw a square. On each side describe an equilateral triangle with the vertices outward. Join the vertices, and show that the figure formed is a square. How long is each side of this square?

8. The diagonals of a parallelogram are 2.4 and 4.2 inches long and contain an angle of 64°. Construct the parallelogram, and measure its sides.

9. Make a square, one diagonal being 3.96 inches. Letter it as in Fig. 28. Bisect *AE*, *EB*, *CE*, and *ED*, and show that the middle points of these lines are at the corners of a square.

10. The diagonals of a rectangle are 1.4 decimeters long and are inclined at an angle of 53°. Draw the rectangle and measure its sides.

PROBLEM 28. To draw a quadrilateral when its four sides are 1.11, .77, 1.67, and 1.3 inches long, respectively, and the angle formed by the first and second lines is 101° (Fig. 32).

Make *AB* a line of 1.11 inches and *ABC* an angle of 101°. Cut off *BC* equal to .77 inch. With *C* as centre, and with a radius equal to 1.67 inches describe the arc of a circle. With *A* as

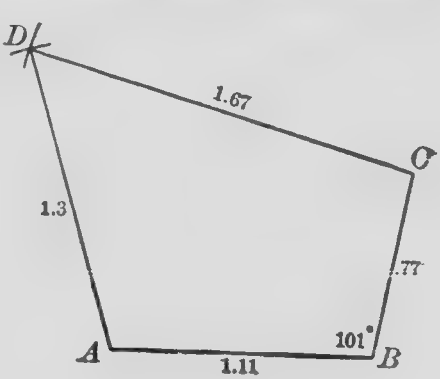


FIG. 32.

centre, and with a radius equal to 1.3 inches describe the arc of a circle. The two arcs meet at *D*. Join *D* to *A* and *C*. Then *ABCD* is the quadrilateral required.

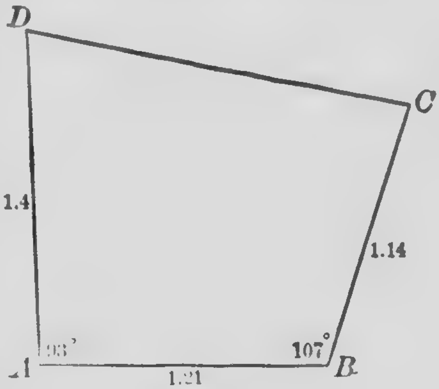


FIG. 33.

PROBLEM 29. The base of a quadrilateral is 1.21 inches; the angles at the base are 93° and 107°, respectively; and the sides adjacent to the base are 1.4 and 1.14 inches. Draw the quadrilateral (Fig. 33).

The method of drawing is easily seen from the accompanying figure.

EXERCISE XIII

1. In Fig. 32, measure the angles at *A*, *C*, and *D*. Their sum added to 101° should be 360°.

2. In Fig. 33, measure *CD*, and the angles at *C* and *D*. The sum of the four angles should be 360°.

3. Lettering a quadrilateral as in Fig. 33, take *AB*, 2.09 inches, *BC*, 1.78 inches, and angles at *A*, *B*, and *C*, 85°, 93°, and 117°, respectively. Draw the quadrilateral. Also measure *AD* and *DC*.

4. In the figure of No. 3, bisect *AB* in *E*, *BC* in *F*, *CD* in *G*, and *AD* in *H*. Join *EF*, *FG*, *GH*, and *HE*. Show that *EFGH* is a parallelogram.

5. Draw the quadrilateral *ABCD*, and measure its four angles, when $AB = 2.6$ inches, $BC = 3.1$ inches, $CD = 4.3$ inches, $DA = 3.5$ inches, and $AC = 4.09$ inches.

6. Draw the quadrilateral *ABCD*, when *AB* = 4.1 cm., *AC* = 5.6 cm., *AD* = 6.9 cm., angle *BAC* = 47°, and angle *CAD* = 32°. How long are *BC* and *CD*?

7. Make a quadrilateral *ABCD* when *AB* and *AD* are each equal to 2.67 inches and meet at an angle of 36°, and *BC* and *CD* are each equal to 1.6 inches. How large are the angles at *B*, *C*, and *D*?

8. Take a line 2.35 inches long and on it describe two equilateral triangles. Show that they make up a rhombus. What is the length of the longer diagonal?

9. Draw the figure *ABCD*, when *AB* = 4 inches, *BC* = 2 inches, angle *A* = 62°, angle *B* = 59°, angle *C* = 121°. Bisect *AD* in *E*, and *BC* in *F*. Measure *DC* and *EF*. The length of *EF* should be half the sum of the lengths of *DC* and *AB*.

10. In the figure of No. 9, show that *DC* and *AB* are parallels. What is the distance between them?

NOTE. — The figure in No. 9 is a *trapezium*. Define a trapezium.

CHAPTER V

AREAS

By the *area* of a figure is meant the amount of space within its boundary. The area is expressed in square centimeters, square meters, square inches, square miles, etc. The shaded part of Fig. 34 is a square centimeter.

PROBLEM 30. A rectangle is 5 centimeters long and 3 centimeters wide. Find its area (Fig. 34).

Draw a rectangle of the given dimensions. Divide the length into 5 and the width into 3 equal divisions. Through these divisions draw parallel lines as in Fig. 34. The rectangle is now divided into 15 squares, each equal to a square centimeter. Therefore the area of the rectangle is 15 square centimeters. Notice that 5 × 3 = 15. Hence, the area of a rectangle is equal to the product of its length and width.

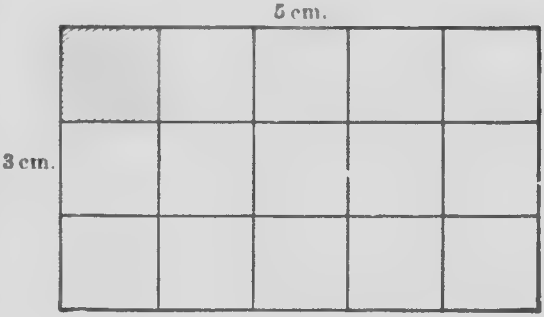


FIG. 34.

PROBLEM 31. A parallelogram is on a base 5 centimeters long and has a perpendicular height of 3 centimeters. Find its area (Fig. 35).

Make a parallelogram *ABCD*, having its base *AB* 5 centimeters long, and its perpendicular height *BE* 3 centimeters long. Erect the perpendicular *AF* and extend *CD* to meet it at *F*. By measurement, *EC* is found to be as long as *FD*, *BE* as long as *AF*, and *BC* as long as *AD*. Therefore, the triangle *BEC* is of the same size as the triangle *AFD*. If we take the triangle *BEC* from the right hand side of the parallelogram *ABCD* and put it on the other side, we have the figure *ABEF*, which must be equal in area to the parallelogram *ABCD*. Now *ABEF* is a rectangle and its area is $5 \times 3 = 15$ square centimeters. Therefore the area of the parallelogram is 15 square centimeters. Hence, the area of a parallelogram is equal to the product of the length of its base and perpendicular height.

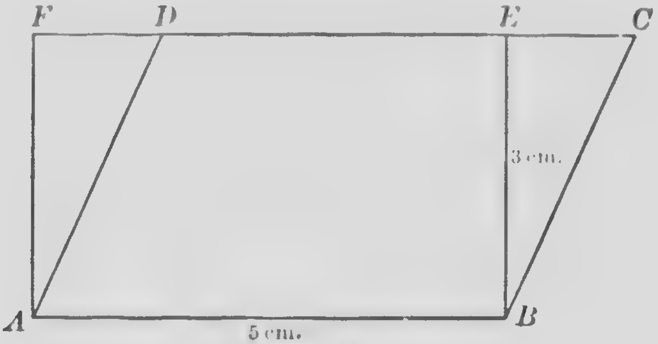


FIG. 35.

NOTE. — The perpendicular height is often called the *altitude*.

PROBLEM 32. A triangle is on a base 4 centimeters long, and has an altitude of 3 centimeters. Find its area (Fig. 36).

Make a triangle ABC, having its base AB 4 centimeters long, and its altitude CD 3 centimeters long. Draw BE parallel to AC and CE parallel to AB. Then $ABEC$ is a parallelogram and its area is 12 square centimeters. By measurement, CE is found to be equal to AB, and EB to CA. Therefore the triangle BEC is equal to the triangle ABC. Now both make up the parallelogram $ABEC$. The triangle ABC is one-half of the parallelogram $ABCD$, and therefore its area = 6 square centimeters. Notice that $6 = \frac{4 \times 3}{2}$. Hence, the area of a triangle is equal to one-half the product of its base and altitude.

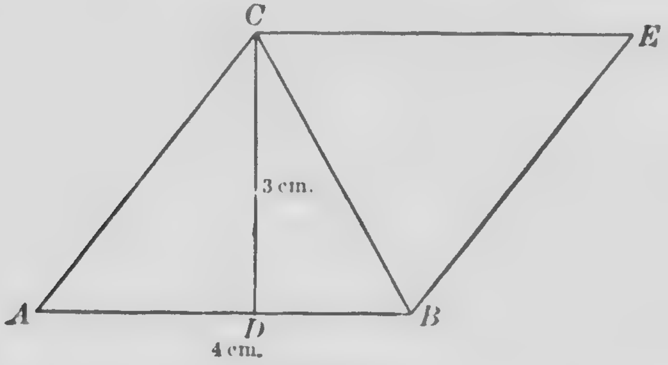


FIG. 36.

PROBLEM 33. A quadrilateral has sides of $1\frac{1}{2}$, 1, $1\frac{1}{4}$, and $1\frac{3}{8}$ inches, and the angle formed by the first and second is 103°. Find its area.

1. Divide the quadrilateral into two triangles by joining D to B. Measure the lengths of DB and the perpendiculars upon it from A and C. Find the area of each triangle. The area of the quadrilateral is equal to the sum of the areas of the triangles.

This method of dividing the figure into triangles may be employed in finding the area of figures bounded by any number of straight lines.

2. The following is another way (Fig. 37): Join *D* to *B*, and draw *CE* parallel to *DB*. Join *D* to *E*. Then the triangle *AED* is equal in area to the quadrilateral *ABCD*. The area of the triangle $= \frac{1.28 \times 2.47}{2} = 1.5808$ square inches. Therefore the area of the quadrilateral $= 1.5808$ square inches.

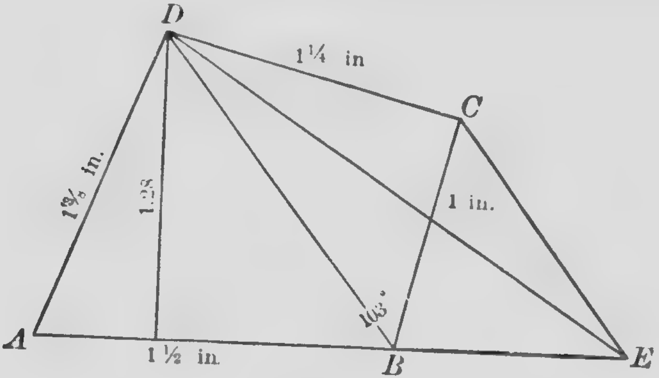


FIG. 37.

This method of reducing the quadrilateral to a triangle of equal area may be extended to find the area of figures of any number of sides. For example, a six-sided figure may be reduced to a five-sided one, and then to a four-sided one, and then to the triangle. See Problem 36.

EXERCISE XIV

1. Draw a square 4 inches on a side. By drawing two sets of parallel lines 1 inch apart, show that the area of the square is 16 square inches.

2. Draw a parallelogram whose two adjacent sides are 4 inches and 3 inches long and meet at an angle of 64°, and find its area.

3. Draw a triangle whose sides are 2.7, 3.4, and 4.1 inches, respectively, and find its area.

4. Multiply 2.7 by the length of the perpendicular on that side from the opposite angle. Similarly, multiply 3.4 and 4.1 by the lengths of the perpendiculars from the opposite angles. The three products should be the same.

5. Draw the quadrilateral $ABCD$, when $AB = 3.2$ inches, $BC = 3.9$ inches, $CD = 4.7$ inches, $DA = 4.3$ inches, and the diagonal $AC = 5.6$ inches; also, find its area by both methods. How closely do the two results agree?

6. Draw a rectangle whose sides are 4 inches and $2\frac{1}{4}$ inches, and draw a square of the same size.

7. $ABCD$ is a trapezium. $AB = 4$ inches, $AD = 2\frac{1}{4}$ inches, $DC = 6$ inches, angle at $A = 90°$, angle at $D = 90°$. Draw the trapezium and find its area.

8. Draw a quadrilateral when one of its diagonals is $3\frac{1}{2}$ inches long and the perpendiculars on it from the opposite angles are $2\frac{1}{2}$ and $2\frac{1}{4}$ inches long. Find its area. Also reduce it to a triangle of equal area and find the area of the triangle. How closely do the two results agree?

9. A trapezium $ABCD$ has $AB = 4.8$ inches, $DC = 3.2$ inches, the perpendicular distance between the parallel sides AB and $CD = 3$ inches, and angle at $A = 54°$. Find its area by reducing it to a triangle of equal area. The answer is 12 square inches. Notice that

$$12 = \frac{4.8 + 3.2}{2} \times 3.$$

Can you deduce the rule for finding the area of a trapezium?

10. A barn has these dimensions: Length = 50 feet, breadth = 40 feet, height of corner-post = 16 feet, and height of gable 31 feet. Draw (1) a rectangle to represent one side of the barn, (2) a rectangle to represent one side of the roof, and (3) a five-sided figure to represent one end of the barn. Find the area of each figure and that of the whole outside surface of the barn.

MISCELLANEOUS EXERCISES

NOTE. — Exercises taken from the examination papers set for entrance to the high schools of Nova Scotia are indicated by the word *Entrance.*

1. Draw parallelogram *ABCD*, making the sides *AB* and *AD* 2 and $1\frac{1}{2}$ inches long, and the angle *BAD* 70°. Measure the other sides and angles.

2. A rectangular lawn has a flower bed in the centre. The lawn is 520 feet long and 240 feet wide. The bed is rectangular and is 12 feet long and 8 feet wide. Draw the diagram on the scale of 100 feet to an inch.

3. The three sides of a triangle are 24, 32, and 40. Construct the triangle and measure its angles. (Entrance, 1893.)

4. A man wishes to find how far a buoy is from the shore. When he is abreast of the buoy it bears directly east. When he has walked 600 feet directly north, he notices that the buoy bears southeast. How far off is the buoy from his first place of observation?

5. Take a line 2.37 inches long and on each side of it describe an equilateral triangle. What is the shape of the quadrilateral formed? Measure its longer diagonal.

6. The foot of a ladder is placed on level ground 27

feet from the side of a house, and the end reaches a window 36 feet from the ground. Find the length of the ladder and its angle of elevation with reference to the level ground. (Entrance, 1894.)

7. Draw a triangle whose sides are 2.3, 1.7, and 2.9 inches. Join the middle points of the sides to the opposite angles.

8. In the centre of a square garden is a square lawn 24 feet on a side. Outside of the lawn is a gravel walk 4 feet wide, and then there is a flower border 6 feet wide. Draw a diagram to represent the lawn, walk, and border.

9. Make an isosceles triangle on a base 2 inches long and with an altitude of 3 inches. Measure the angles.

10. A rectangular field, 300 yards long and 150 yards wide, is divided into four equal parts by two bands of trees, 29 feet wide, and parallel to the sides. Represent this.

11. Draw the floor plan of a schoolroom, as neatly as you can, to the scale of 6 feet to an inch. (Entrance, 1895.)

12. A ship sails 17 miles due east, then 14 miles due north, then 24 miles due northeast, then 28 miles due south. Find its distance from the point of starting, and the angle made with the north and south line by the line of distance. (Entrance, 1895.)

13. *ABCD* is a field. $AB = 400$ feet, $BC = 500$ feet, $CD = 550$ feet, angle at $B = 110°$, angle at $C = 80°$. Draw the field and reduce it to a triangle of equal area. Find the area of the triangle.

14. The base of a triangle is 2.67 inches and the angles at the base are 62° and 57°. Construct the triangle and measure the other two sides.

15. Construct a square *ABCD* on a line 4 inches long.

In AB take a point D 1 inch from A; in BC, G 1 inch from B; in CD, F 1 inch from C; and in AD, E 1 inch from D. Join DG, GF, FE, and ED. Show that $DEFG$ is a square.

16. From a point A in a level plain a straight line 432 feet long was measured to the east end of an impassable swamp, and another line making an angle of 68° with the first line at A was measured 367 feet to the west end of the swamp. Find by construction the length of the swamp from the said east point to the said west point. (Entrance, 1898.)

17. Two adjacent sides of a quadrilateral are $2\frac{1}{2}$ and $3\frac{1}{4}$ inches long, and the included angle is 65°. The other sides are 3 and 4 inches. Construct the figure. Show that the middle points of the sides are at the corners ot a parallelogram.

18. Two straight roads meet at an angle of 45° and distances along them are measured from the common point. How far is the 10th milestone on the one road from the 15th milestone on the other?

19. Two perpendicular trees, one 60 feet high and the other 37 feet high, are standing 55 feet apart. What is the distance between their tops? (Entrance, 1901.)

20. A man walks 3 miles, turns through 45° to his left, goes half a mile, turns through 90° to his left, and goes 1 mile farther. How far is he from the starting-point?

NOTE. — When he turns through 45°, he turns through an angle of 45° from the direction in which he would have gone if he had not turned. You produce the line beyond the point of turning, and the new direction will make an angle of 45° with the produced part of the line.

21. A ship sails east from A 124 miles to B, then turns 30° towards the south and sails 115 miles to C, then turns 60° more to the south and sails 156 miles to D. How far is D from A in a straight line, and what angle does AD make with AB? (Entrance, 1904.) See note under Ex. 20.

22. Two vertical poles are 50 feet apart and their tops are 70 feet apart. The shorter pole is 15 feet high. What is the length of the longer pole?

23. BC is a tower standing on level ground. A is a point 320 feet from the base of the tower. The angle BAC is 23°. How high is the tower?

24. Draw a diagram of your playground, marking on it the flower beds, ornamental trees, etc.

25. The hands of a clock are 5 and $3\frac{3}{4}$ inches long respectively. What is the distance between their extremities at 2 o'clock? At 3.30 o'clock? At 9 o'clock?

26. A tree 48 feet high casts a shadow 30 feet long. How many degrees is the sun above the horizon?

27. If the sun is 40° above the horizon, how long a shadow will be cast by a vertical pole 40 feet high?

28. A gun having a range of 5 miles fires upon a ship bearing southeast from the gun, and 3 miles distant from it. Thereupon the ship sails in a southwest direction. How far must it go in this direction so as to reach a point just out of range of the gun?

29. Take a line 4 inches long and one 2 inches long, and by means of the protractor describe an equilateral triangle on each. Find the areas of the triangles. How many times greater is the former than the latter?

30. A vessel sails 30 miles due west: it then turns north-

east and sails till it reaches a point due north of the point left, and then sails directly for this point. How far did it sail all together?

31. *ABCD* is a quadrilateral field. *B* is due west from *A*; *C*, northeast from *B*; *D*, due east from *C*; *AB* is 450 yards long; *BC*, 400 yards; and *CD*, 500 yards. Plot the field on a scale of 100 yards to an inch, and find its area.

32. A railway runs 2 miles direct from *A* to *B* and then turns to the left through an angle of 60° and goes straight on to *C* for 3 miles. Draw a diagram, 2 inches to a mile, and measure the distance from *A* to *C* direct.

33. Make a diagram of a four-sided field from the following data: A surveyor walks along one side for a distance of 320 yards; then he turns to the right 80° and walks along the second side for a distance of 440 yards; then he turns to the right 90° and walks along the third side for a distance of 360 yards. What is the length of the fourth side of the field? What is its area?

34. Take a line *AB* 4.37 inches long and bisect it in *C*. Through *C* draw a line *DE* at right angles to *AB*. Make *DC* 2.06 inches and *EC* 1.97 inches. Show that $AE = BE$ and $AD = DB$.

35. A man, standing 40 yards from the foot of a tower, finds that the top of the tower is 20° above the level of the eye; and in making the observation his eye is 5 feet above the level of the ground. How high is the tower?

36. Northwest from my house and 423 yards from it is an oak tree; southwest from the tree and 379 yards from it is a well. Draw a plan, and find how far the well is from the house.

37. The wire stay of a telegraph pole is fastened to the top of the pole and to a point in the ground 8 feet from its foot, and makes an angle of 65° with the ground. Find the length of the wire and the height of the pole.

38. To measure the breadth of a river a man proceeded thus: He placed a pole *A* on one bank exactly opposite to a tree *B* on the other bank; he then walked 60 paces to *C* so that *AC* was at right angles to *AB*; and at *C* he measured the angle *ACB* and found it to be 50°. What is the breadth of the river? (A pace is 30 inches.)

39. *A* and *B* are two forts 8 miles apart, *A* being due north of *B*. An opposing force is encamped at *C* whose direction from *A* is S. 35° E. and from *B* N. 20° E. How far is the force from the nearer fort? (By *C* being S. 35° E. from *A* is meant that *C* is towards the south and that *CA* makes an angle 35° with the north and south line through *A* and on the east side of it. The angle *CAB* is therefore 35°.)

40. A cross-bar is fixed on the tops of two posts which are 16 feet high and 10 feet apart. A swing 2 feet wide, 3 feet from the ground, and midway between the posts is fastened to the bar. Make a diagram to represent this.

PART II

CHAPTER VI

POLYGONS

NOTE. — *Polygons* have more than four sides. *Regular* polygons have equal sides and equal angles. The size of an angle of a regular polygon can be got by the following rule: Divide 360 by the number of sides, and subtract the quotient from 180. The remainder gives the number of degrees in the angle. For example, a decagon has 10 sides. Each angle of a regular decagon is determined thus: 360 ÷ 10 = 36. 180 – 36 = 144. Therefore each angle = 144°.

PROBLEM 34. To describe a regular pentagon on a base, 1.12 inches long, and to find its centre (Fig. 38).

A pentagon has 5 sides. 360 ÷ 5 = 72. 180 – 72 = 108. Each angle is 108°.

Take *AB*, a line 1.12 inches long, and at *A* and *B* make angles of 108° each. Make *AE* and *BC* each 1.12 inches long. At *E* and *C* make angles of 108°. *ED* and *CD* intersect at *D*. Then *ABCDE* is a regular pentagon. To find the centre, bisect the angles at *A* and *B* by lines meeting at *F*. Then *F* is the centre.

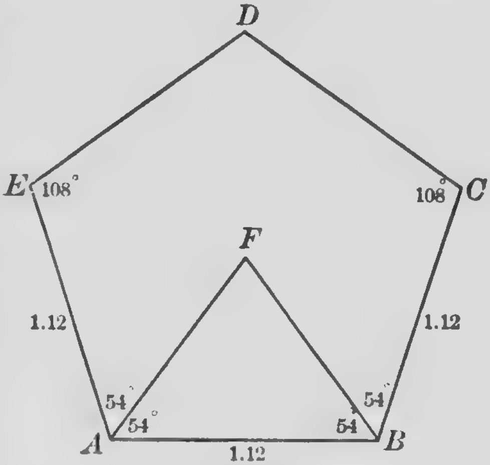


FIG. 38.

EXERCISE XV

(On Fig. 38)

1. Measure *ED*, *CD*, and the angle at *D*.

2. Bisect the angle at *F*. Measure the length of the bisector from *F* to where it cuts *AB*, and show that it bisects *AB*, and forms an angle of 90° with it. How large is the angle *AFB*?

3. Find the area of the triangle *FAB*. How many triangles equal to *FAB* will make up the whole pentagon? What, then, is the area of the pentagon?

4. Join *AC* and *BD*. Call the point of intersection of these lines *H*. Measure the lengths of *AH*, *HC*, *DH*, and *HB*. The number expressing the length of *AC* multiplied by that expressing the length of *HC* should be equal to the square of the number expressing the length of *AH*.

5. Measure the distances of *F* from *A*, *B*, *C*, *D*, and *E*.

6. What is the perpendicular distance of *F* from each of the sides?

7. How far is *A* from *C*? *A* from *D*? *B* from *E*? *B* from *D*? *C* from *E*?

PROBLEM 35. To describe a regular hexagon on a base, 1 inch long, and to find its centre (Fig. 39).

A hexagon has six sides. $360 \div 6 = 60$. $180 - 60 = 120$. Each angle, therefore, is 120°.

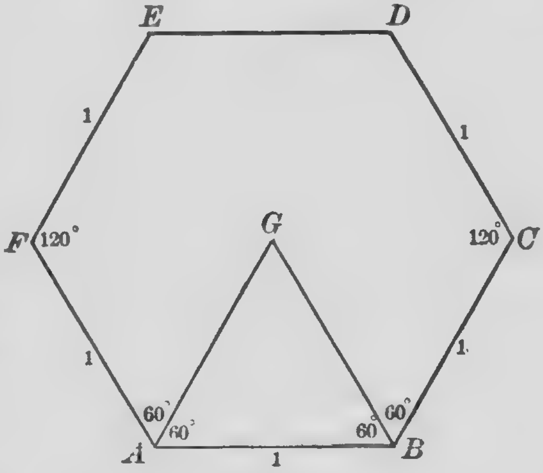


FIG. 39.

Take *AB*, a line 1 inch long. At *A* and *B* make angles of 120° each. Make *AF* and *BC* each 1 inch long. Make angles at *F* and *C* 120° each and *FE* and *CD* each 1 inch long. Join *ED*. Then *ABCDEF* is a regular hexagon.

To find the centre, bisect the angles at *A* and *B* by lines meeting at *G*. Then *G* is the centre of the hexagon.

EXERCISE XVI

(On Fig. 39)

1. Measure *ED* and the angles at *E* and *D*.

2. How far is *G* from each of the angles *A*, *B*, *C*, *D*, *E*, and *F*?

3. What is the perpendicular distance of *G* from each of the sides?

4. What is the area of the triangle *AGB*? What is the area of the hexagon?

5. Join *GD*. Show that *GD* bisects the angle at *D*, and is in a straight line with *AG*.

6. Show that the hexagon has three pairs of parallel sides.

7. Join *E* to *A* and *D* to *B*. Show that the figure *EABD* is a rectangle.

8. Join *G* to *C* and show that the figure *GABC* is a rhombus.

9. Join *B* to *F*. Show that *BF* and *AG* bisect one another at angles of 90°.

10. Produce *FA* and *CB* to meet in the point *H*. Show that the triangle *AHB* is equilateral.

PROBLEM 36. To make a pentagon $ABCDE$, when $AB = 1.3$, $BC = 1.16$, $CD = 1.05$, $DE = 1.32$, and $EA = 1.12$ inches; and the angle $ABC = 113°$, and the angle $BCD = 101°$. And to reduce the pentagon to a triangle of equal area (Fig. 40).

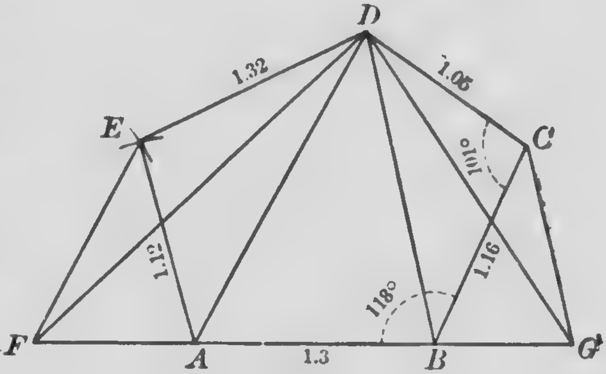


FIG. 40.

Take AB 1.3 inches long, and BC 1.16 inches long, and CD 1.05 inches long. Place AB and BC to make an angle of 113°, and CD and BC to make an angle of 101°. With D as centre and with a radius of 1.32 inches describe the arc of a circle. With A as centre and with a radius of 1.12 inches describe the arc of a circle. The two arcs intersect in E. Join E to A and D. Then $ABCDE$ is the pentagon required.

To find a triangle of equal area proceed thus: Join DB and DA. Measure the angle DBC and make the angle BCG equal to it. In other words, draw CG parallel to DB. Also draw EF parallel to DA. Join DF and DG. Then the triangle DFG is equal in area to the pentagon $ABCDE$.

EXERCISE XVII

1. Make a regular octagon on a base .79 inch long.

2. Make a nonagon on a base .79 inch long. Which takes up more space on your paper, the octagon or the nonagon?

3. What is the sum of all the angles of the pentagon in Fig. 38? In Fig. 40 how large are the angles *EAB*, *AED*, and *EDC*? What is the sum of all the five angles?

4. Make a pentagon *ABCDE*, when the following are given: $AB = 2$ inches, angle $ABE = 26°$, angle $BAE = 84°$, angle $BED = 76°$, angle $EBD = 28°$, angle $BDC = 41°$, and angle $DBC = 30°$. Measure all the other sides.

5. Draw the pentagon when the angles at *A*, *B*, *C*, and *D* are respectively 90°, 101°, 107°, and 123°, and the base $AB = 1.91$ in., $BC = 1.37$ in., and $CD = 1.64$ in. Measure the other sides.

6. Make a triangle equal in area to the pentagon in No. 5.

7. Construct the pentagon when the angles at *A* and *B* are 90° and 110° respectively, and *AB*, *BC*, *CD*, *DE*, and *EA* are respectively 2, $2\frac{1}{8}$, $2\frac{3}{8}$, $2\frac{1}{2}$, and $2\frac{3}{4}$ inches long.

8. Take a point *O* and from it draw six lines, *OA*, *OB*, *OC*, *OD*, *OE*, and *OF*, whose lengths are 1.6, 2.3, 3.4, 1.9, 2.7, and 3.1 inches respectively, each of the angles *AOB*, *BOC*, etc., being 60°. Join *AB*, *BC*, *CD*, *DE*, and *EF*. Measure these lines.

9. Draw the pentagon *ABCDE* when $AB = 250$, $AC = 300$, $AD = 200$, and $AE = 150$ yards: angle $BAC = 32°$, angle $CAD = 40°$, and angle $DAE = 29°$. Measure the lengths of *BC*, *CD*, and *DE*. Scale 100 yards to an inch.

10. In the figure to No. 9, reduce the quadrilateral *ABCD* to a triangle of equal area. What is the area?

CHAPTER VII

PRACTICAL APPLICATIONS

THE *angle of elevation* of one point above another is the angle formed at the lower point by a horizontal line and a line sloping upward and passing through the higher point.

The *angle of depression* of one point below another is the angle formed at the higher point by a horizontal line and a line sloping downward and passing through the lower point.

PROBLEM 37. At a horizontal distance of 215 feet from the bottom of a tower, the angle of elevation of its top was observed to be 25°. Find the height of the tower. Scale, 100 feet to an inch (Fig. 41).

CB represents the tower; *A*, the point of observation; *AB*, the horizontal distance; and *CAB*, the angle of elevation of *C* above *A*.

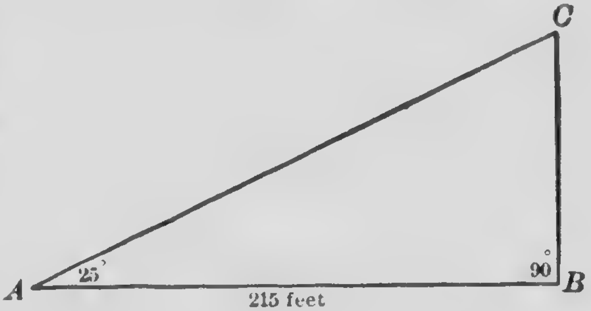


FIG. 41.

The method of construction is easily seen from the accompanying figure.

Measure *CB* and neatly record its length.

Problem 38. A tower stands in a level plain. At a point in the plain the angle of elevation of the top of the tower is 25°, and at a point 190 feet nearer to the tower, the angle of elevation of the top of the tower is 69°. Find the height of the tower. Scale, 100 feet to an inch (Fig. 42).

Take a point *A* and from it draw a horizontal *AD*, 1.9 inches long. At *A*, make an angle of 25°, and at *D*, an angle of 69°, as in the accompanying figure. Produce the arms of these angles to meet at *C*. *C* represents the top of the tower. Draw *CB* to make an angle of 90° with *AD* produced. Then *CB* represents the tower. Measure its length and neatly enter it in your diagram.

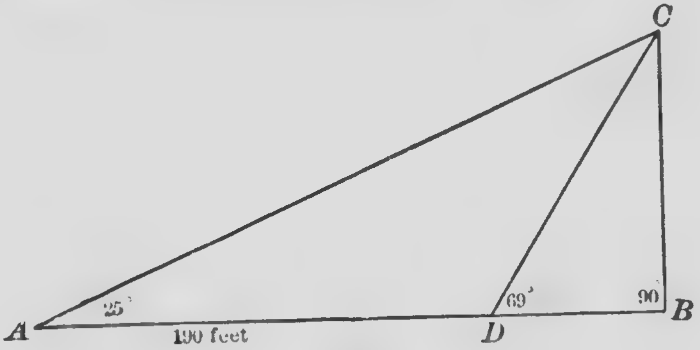


Fig. 42.

Problem 39. From the summit of a lighthouse 62 feet high, standing on a rock, the angle of depression of a ship is 46°; and at the bottom of the lighthouse, the angle of depression is 35°. Find the horizontal distance of the ship and the height of the rock. Scale, 100 feet to an inch (Fig. 43).

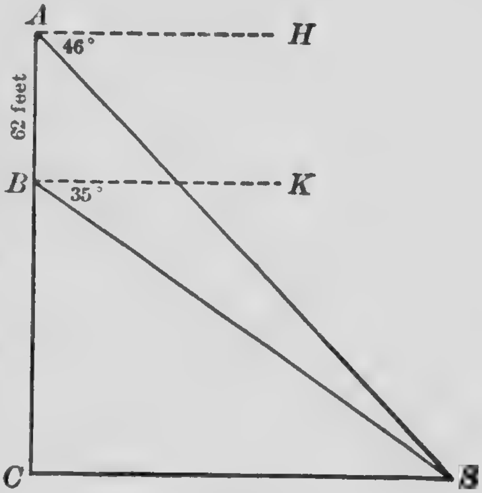
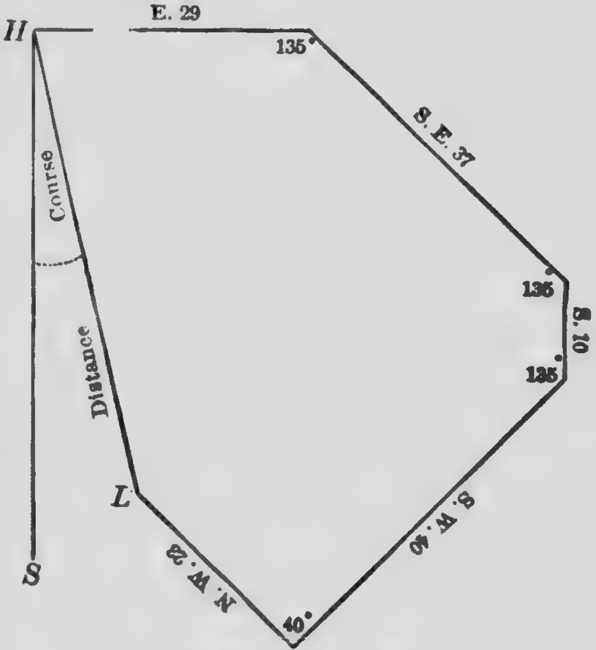


FIG. 43.

Take *AB*, a vertical line .62 inch in length. Draw *AH*, a horizontal line through *A*, and make the angle *HAS* 46°. Similarly make the angle *KBS* 35°. Produce the arms of these angles to meet at *S*. *S* represents the ship. From it draw *SC*, a horizontal line. Produce *AB* to meet *SC* in *C*. *SC* represents the horizontal distance of the ship, and *BC* the height of the rock. Measure these lines and enter their values in your diagram.



Scale 25 m. = 1 in.

FIG. 44.

PROBLEM 40. Suppose a vessel to leave Halifax and upon the following courses and distances: east 29 miles; southeast 37 miles; south 10 miles; southwest 40 miles, and northwest 23 miles: find the direct course and distance from Halifax to the last position of the ship (Fig. 44).

The method of construction is easily seen from the accompanying figure. *H* is the point left, *L* is the point reached, *HS* is the north and south line through *H*, angle *LHS* is the direct course, and *LH* is the distance.

Ans. Course is S. 13° E.; distance is $49\frac{1}{3}$ miles nearly.

NOTE. This is a problem in Plane Sailing. In it we regard the sea as having a flat and not a curved surface.

PROBLEM 41. To find the area of a field shaped like Fig. 45, a land surveyor proceeds thus: He measures the distance from *A* to *D*, and takes points in *AD* so that the lines joining them to the corners of the field make angles of 90° with *AD*. Then he measures the distances *AG*, *AH*, *AK*, *AL*, and *AD*, which are respectively 180, 360, 464, 684, and 840 yards. He also finds $BG = 232$ yards, $FH = 256$ yards, $CK = 260$ yards, and $LE = 164$ yards. How many acres are there in the field? Scale is 400 yards to an inch in Fig. 45.

The student should take a scale of 200 yards to an inch. Draw *AD* the required length. Measure off distances of 180, 360, etc. Draw the perpendiculars *GB*, *HF*, etc. Join *AB*, *BC*, etc. Thus you have the diagram drawn.

The figure is divided into triangles and trapeziums. Find the area of

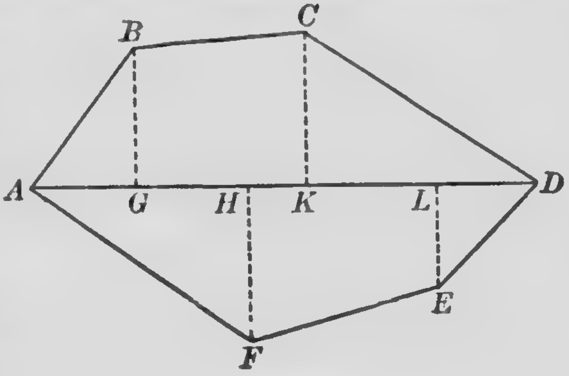


FIG. 45.

each. To find the area of the trapezium *BGKC*, add 232 to 260; take half the sum; and multiply the quotient by the length of *GK*.

EXERCISE XVIII

1. A flagstaff stands on the top of a tower. The tower itself is on level ground. At a certain point on the ground the angle of elevation of the top of the staff is 43°, while that of the top of the tower is 37°. At a point 200 feet farther away from the tower the angle of elevation of the top of the staff is 31°. How high is the staff?

2. A rock is 483 feet high above a plain. A tower stands on the rock. The angles of depression of a point in the plain are 52°, when observed from the top of the tower; and 48°, when observed from the top of the rock. Find the height of the tower.

3. From the top of a hill 360 feet high above a plain, the angles of depression of the top and bottom of a tower standing on the same plain are 40° and 51°, respectively. Find the height of the tower.

4. A vessel sails 17 miles due south, then 21 miles due southwest, then 18 miles due northwest, and then 13 miles due west. Find the direct course and distance from the place left to that finally reached.

5. A kite is flying to the full length of its string, namely, 125 yards, and the string makes an angle of 45° with the horizontal. How high is the kite?

6. A tower is 160 feet high. At a certain point on the ground the angle of elevation of its top is 36°. At the same point the angle of elevation of the top of a flagstaff

standing on the tower is 44°. Find the distance of the point of observation from the tower and the length of the flagstaff.

7. Find the area of the figure $ABCDE$ from these measurements: $AB = 20$ chains, $AC = 18$ chains, $AD = 25$ chains, $AE = 30$ chains, angle $BAC = 45°$, angle $CAD = 43°$, angle $DAE = 41°$. What is the length of each of the other sides?

8. Two ships C and D are at anchor within sight of the shore. The distance between them is got thus: A base line AB of 400 yards is taken. The observer notices that the angle BAC is 60°, the angle BAD is 40°, the angle ABC is 50°, and the angle ABD is 90°. How far apart are the ships?

9. Find the area of a field $ABCDE$ from these measurements: AD is 600 yards, and along it are measured AX 230 yards, AZ 350 yards, and AY 470 yards. At X and Y are drawn XB 240 yards and YC 160 yards, on the same side of AD and perpendicular to it; and at Z is drawn ZE 220 yards on the other side of AD and perpendicular to it.

10. A tower 50 feet high stands on the edge of a cliff, and the angle of depression of a boat as seen from the top of the tower is 20°, and from the bottom 14°. Determine the height of the cliff and the distance of the boat from its foot.

CHAPTER VIII

SCALES

DRAWINGS are seldom as large as the objects which they represent. The map of a district or the drawing of a large piece of machinery may be made on a space a few square inches in area. Every line or part of the object is reduced in the same proportion, so that the drawing is a true representation of the original. Such drawings are made *to scale.*

There are several ways of expressing the scale. Sometimes it is constructed at the foot of the drawing, as is usually done in the case of maps. At other times it is expressed in words thus: "The representative fraction = $\frac{1}{240}$," meaning that each line on the diagram is $\frac{1}{240}$ of its true length; or "1 inch = 100 feet" or "100 feet to an inch," meaning that a line 100 feet long is represented in the drawing by a line one inch in length. Whatever be the method of expressing the scale, it is usually represented at the foot or side of the drawing.

When the scale is expressed in words, the one way may be changed into the other. If the representative fraction $= \frac{1}{240}$, then 1 foot = 240 feet; or, if 1 inch = 100 feet, then the representative fraction $= \frac{1}{1200}$, for 100 feet = 1200 inches.

The scale most convenient to adopt in any particular case will depend on the size of the object, the size of your paper, and the purpose which the drawing is to serve. For example, the representative fraction of the plan of a field containing a few acres will be much larger than that of the map of the county. Or, again, an ordinary map of a certain district will be smaller than that published for military purposes, will contain fewer details, and hence be on a smaller scale.

When the object is small, it is best to make the drawing on an enlarged scale. For instance, the mechanism of a watch can be best understood when the drawing is considerably larger than the watch itself.

PROBLEM 42. To construct a scale of equal parts (Fig. 46).

Take a line of any length and on it lay off any number of equal divisions, *AB*, *BC*, *CD*, etc. In Fig. 46 each division is one inch long. By trial, divide *AB* into ten equal divisions. The construction is plainly seen from the figure. If *AB*, *BC*, *CD*, etc., measure inches, the divisions of *AB* measure tenths of inches. Then, *Da* is 2.4 inches long. If the large divisions represent 10 inches each, the smaller divisions represent inches. Then *Da* represents a line 24 inches long. Again, if the large divisions represent 100 inches each, the smaller divisions stand for 10 inches, and *Da* represents a line 240 inches long. And so on. Thus the same scale will measure lengths greatly differing from one another.

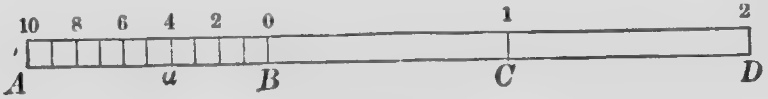


FIG. 46.

Problem 43. To construct a scale of $\frac{1}{24}$, or 1 inch = 2 feet, and long enough to measure 6 feet (Fig. 47).

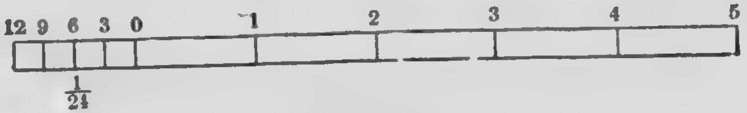


Fig. 47.

Since 2 feet is to be represented by 1 inch, 6 feet will be represented by 3 inches.

Take a line of this length and by trial divide it into 2 equal parts and each of these parts into 3 equal parts. The whole line is now divided into 6 equal parts. Number the divisions as in the figure and divide the division on the left into 4 equal parts.

This scale will measure feet up to 6, and inches at intervals of three.

Problem 44. To draw a scale of $\frac{1}{84480}$, or 1 inch = $1\frac{1}{3}$ miles, to measure miles and furlongs, and long enough to measure 4 miles (Fig. 48).

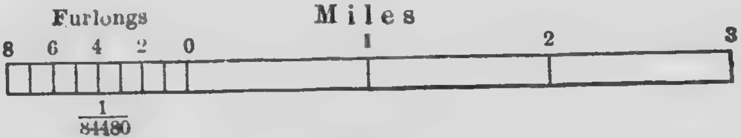


Fig. 48.

Since $1\frac{1}{3}$ miles = 1 inch, 4 miles = 3 inches. Take a line of this length and divide it into 4 equal parts, and number them as in the figure. Divide the division on the left into 8 equal parts. Each of them represents a furlong.

This scale will measure miles and furlongs up to 4 miles.

Problem 45. To make a scale of yards of $\frac{1}{360}$, and a comparative scale of meters, long enough to measure 30 yards and 30 meters respectively (Figs. 49, 50).

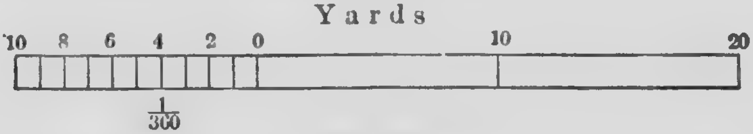


Fig. 49.

30 yards $= 30 \times 36$ inches $= 1080$ inches. $\frac{1}{360}$ of $1080 = 3$. 30 meters $= 30 \times 39.37$ inches $= 1181.1$ inches. $\frac{1}{360}$ of $1181.1 = 3.28$. The scales are 3 and 3.28 inches in length and are drawn as shown in the figures.

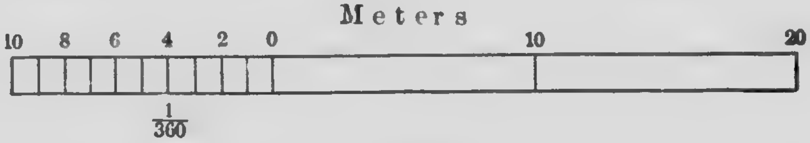


Fig. 50.

Note.—Test the correctness of the scales by an arithmetical calculation. For example, measure 17 meters on the second scale and determine the value in yards on the first scale. You should get 18.6 yards, for 17 meters $= \frac{17 \times 39.37}{36}$ yards $= 18.6$ yards nearly.

Problem 46. To make a diagonal scale to measure yards and feet, and long enough to measure 12 yards.

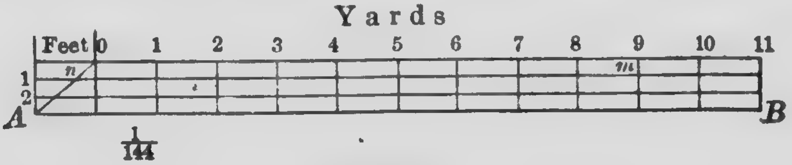


Fig. 51.

The representative fraction is $\frac{1}{144}$ (Fig. 51).

$\frac{1}{144}$ of 12 yards $= 3$ inches. Take a line AB of this length and divide it into 12 equal parts. From *A* draw a line at right angles to *AB* and on it set off three equal divisions. At every division draw a line parallel to *AB* and complete the scale as in the figure. *nm* is read "9 yards 1 foot." Take off the length 8 yards 2 feet.

NOTE.—The diagonal scale on your ruler is constructed by a method similar to that of Problem 46. 5.86, 63.4, and 472 are examples of numbers that can be taken off from the diagonal scale. In 5.86, the large divisions stand for units; in 63.4, for tens; and in 472, for hundreds.

EXERCISE XIX

1. By means of the scale of Fig. 46 draw a line 1.8 inches or units long.

2. By means of the scale of Fig. 47 draw a line to represent 4 feet 9 inches.

3. By means of the scale of Fig. 48 draw a line to represent 3 miles 3 furlongs.

4. By means of the scale of Fig. 49 draw a line to represent 18 yards.

5. By means of the scale of Fig. 50 draw a line to represent 15 meters.

6. By means of the scale of Fig. 51 draw a line to represent 7 yards 2 feet.

7. This line, ______________________________, represents different measures according to the scales used. What are they with reference to the scales in Figs. 46–51 respectively?

8. By means of scales convert 22 yards to meters and 19 meters to yards. See how your values agree with those got by an arithmetical calculation.

9. Express this otherwise: "The scale is 4.4 inches = $\frac{3}{4}$ mile."

10. A country is 360 miles long and 240 miles wide. A map of it is to be put on a sheet of paper no longer than this page. What will be a convenient scale to adopt?

PROBLEM 47. To construct a scale of chords (Fig. 52). Describe the quadrant *CAB*. With *C* as centre and *CA* as radius describe an arc cutting the arc *BC* at the point marked 60°. Similarly with *B* as centre and *BA* as radius an arc can be drawn to cut *BC* at 30°. Thus the arc *BC* is trisected at 60° and 30°. By trial, trisect each of these divisions. The arc is now divided into 9 equal parts of 10° each.

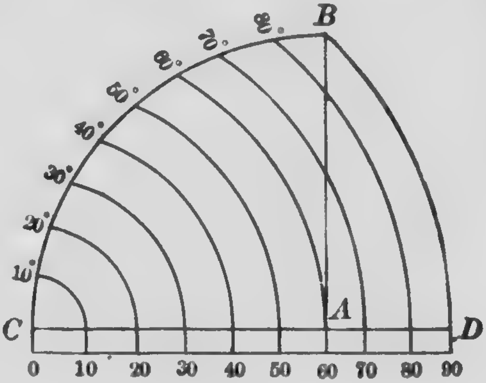


FIG. 52.

With *C* as centre, describe arcs from these divisions, meeting *CD* as shown in the figure. *CD* is the required scale of chords.

NOTES. — 1. This scale has divisions at intervals of 10 and they gradually decrease in length from 0° to 90°. The scale of chords on your rulers has every division from 0 to 90.

2. The distance 0 to 60 is equal to the radius of the quadrant from which the scale of chords was made.

PROBLEM 48. By means of the scale of chords, to construct any angle (Fig. 53).

1. An angle less than 90°, for example 40°.

Take a line *AB*, and, with *A* as centre and with a radius equal to the distance from 0 to 60 on the scale of chords, describe an arc *BD*. With *B* as centre and with a radius

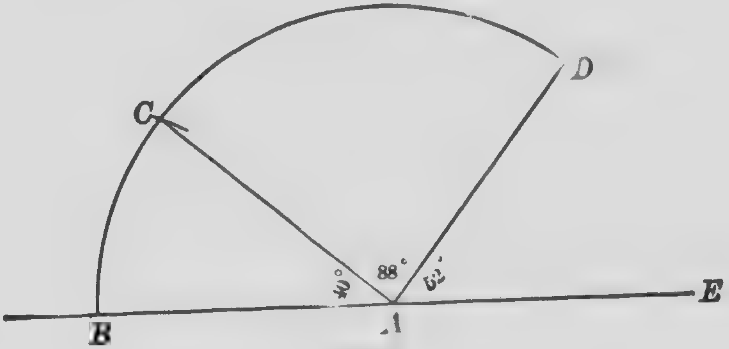


FIG. 53.

0 to 40 on the scale of chords, cut the arc at *C*. Join *CA*. Then *CAB* is an angle of 40°.

2. An angle greater than 90°, for example 128°.

Make the angle of 40° as before. Then with *C* as centre, and with a radius 0 to 88 on the scale of chords, cut the arc *BD* at *D*. Join *DA*. Then *DAB* is an angle of 128°.

Or, produce *BA* to *E* and make *DAE* an angle of 52° (by first method). Then *DAB* = 180° − 52° = 128°.

NOTE. — Instead of 40 and 88, you can use 55 and 73, or any other values less than 90° and together equal to 128°.

PROBLEM 49. By means of the scale of chords, to measure an angle (Fig. 53).

1. An angle less than 90°.

With *A* as centre and with a radius equal to the distance from 0 to 60 on the scale of chords, describe an arc, cutting the arms of the angle at *B* and *C*. Take the distance from *B* to *C* and place it on the scale of chords. It measures 40. Then the angle is 40°.

2. An angle greater than 90°.

Divide the angle into any two parts each less than 90°. *CAB* measures 40° and *DAC* is found to measure 88°. erefore the angle *DAB* measures 128°. Or, produce *BA* to *E* and measure the angle *DAE*. It is 52°. Therefore the angle $DAB = 180° - 52° = 128°$.

EXERCISE XX

1. Draw a scale of $1\frac{1}{4}$ inches = 1 foot, to measure feet and inches. What is the representative fraction?

2. Construct a scale to show yards and feet, on which $3\frac{1}{2}$ inches represent 8 yards. Make it long enough to measure 12 yards. What is the representative fraction?

3. Make a diagonal scale to show miles and furlongs, on which $\frac{3}{4}$ inch represents 1 mile, and long enough to measure 6 miles. On the scale indicate 4 miles 7 furlongs. What is the representative fraction?

4. Construct a scale of $\frac{1}{468}$ long enough to measure 65 yards at intervals of 5 yards.

5. With a radius of $2\frac{1}{2}$ inches describe a quadrant and make a scale of chords to measure angles from 0° to 90° at intervals of 10°.

6. With the scale of No. 5 describe angles of 70°, 90°, and 130°. Use a protractor to test the correctness of the work.

7. Make a triangle whose sides are 3.4, 4.6, and 5.2

inches respectively, and with the scale of No. 5 measure the angles.

8. With a protractor make angles of 20°, 80°, and 130°, and with the scale of No. 5 test your work.

9. On a plan $12\frac{1}{2}$ yards occupy $2\frac{1}{4}$ inches. Draw a scale to indicate yards at intervals of 5 and long enough to measure 30 yards. What is the representative fraction?

10. Draw a line 4.76 inches long to represent 3 feet. Divide the line into 3 equal parts, and the left-hand division into 3 equal parts. Indicate 2 feet 2 inches on the scale.

CHAPTER IX

THE LOCATION OF POINTS

TAKE a horizontal line, *AOX*, and a vertical line, *YOB*, intersecting at *O* (Fig. 54). *AOX* is spoken of as the *axis of X*, *YOB* as the *axis of Y*, and *O* as the *origin*.

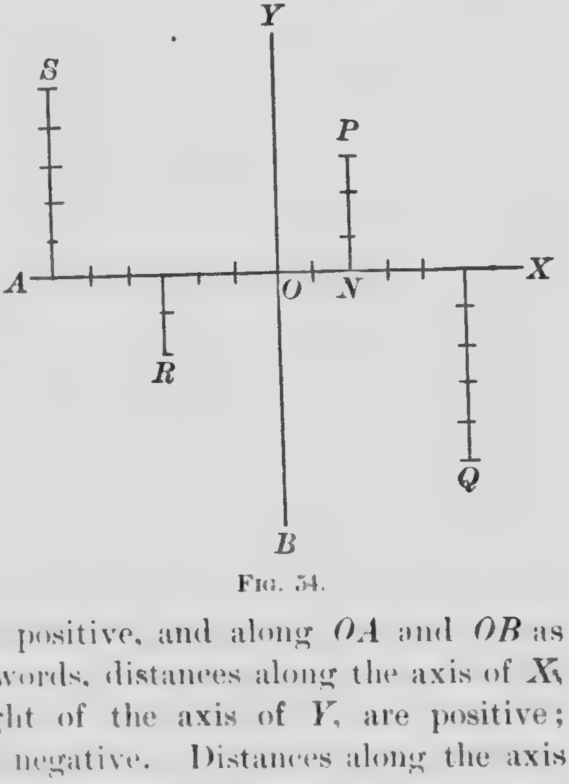


FIG. 54.

By the intersection of the lines four right angles or quadrants are formed. The space *YOX* is the first quadrant; *YOA*, the second; *AOB*, the third; and *BOX*, the fourth.

Distances measured along *OX* and *OY* are regarded as positive, and along *OA* and *OB* as negative. In other words, distances along the axis of *X*, measured to the right of the axis of *Y*, are positive; measured to the left, negative. Distances along the axis of *Y*, measured above the axis of *X*, are positive; measured below, negative.

The location of a point is determined by its distances from the two axes. These distances are known as the *coördinates* of a point.

The distance of a point from the axis of X is called the *ordinate* of the point, and its distance from the axis of Y is called the *abscissa* (plural, *abscissæ*).

In Fig. 54, P is the point (2, 3), because its abscissa ON is 2 units in length and measured to the right of the axis of Y; and its ordinate PN is 3 units in length and measured above the axis of X. S is the point $(-6, 5)$, for it is 6 units to the left of the axis of Y, and 5 units above the axis of X. R is the point $(-3, -2)$, for it is 3 units to the left of the axis of Y, and 2 units below the axis of X. Q is the point $(5, -5)$, for it is 5 units to the right of the axis of Y, and 5 units below the axis of X.

The unit of length may be any convenient one. That of Fig. 54 is $\frac{1}{5}$ inch. That of *squared* paper is usually $\frac{1}{10}$ inch. Squared paper has horizontal lines ruled upon it, $\frac{1}{10}$ of an inch apart. These are crossed by vertical lines, which are $\frac{1}{10}$ of an inch apart. The paper, then, has a large number of small squares. To see the appearance and use of squared paper, the student is referred to Hall and Knight's "Algebra," eighth edition, Chapter XLIV.

In naming a point, the abscissa is mentioned before the ordinate. Thus the point (2, 3) means that 2 is the abscissa of the point and 3 the ordinate.

To determine the location of a point, or, in other words, to *plot* a point, for example the point (2, 3), we proceed thus: measure ON 2 units in length along OX; at N erect a perpendicular, NP, and make it 3 units long. Then P is the point (2, 3).

EXERCISE XXI

(NOTE. — Make the units of length ⅕ inch each.)

1. Plot the points (2, 4), (– 6, – 8), (3, – 5), and (– 2, 6).

2. P is the point (– 2, 8). Q is the point (6, 14). Find the length of PQ. *Ans.* 10.

3. A is the point (0, 6). B is the point (8, 0). Find the length of AB, the coördinates of the middle point of AB, and the distance of this point from the origin.
Ans. 10, (4, 3), and 5.

4. Show that the distance between the points (6, 3) and (– 3, – 9) is 15.

5. Plot the points (6, 0), (0, 6), and (2, 4), and show that they are in the same straight line.

6. Show that the line joining the points (0, 3) and (3, 0) passes through the point (– 2, 5).

7. Plot the points (4, 2), (8, 6), (4, 10), and (0, 6), and show that they are at the corners of a square.

8. Join the point (6, 7) to the points (8, 3) and (– 2, 3), and show that the angle between the two lines is 90°.

9. Show that the line joining the points (4, 2) and (10, 8) is parallel and equal to the line joining the points (2, 8) and (8, 14).

10. A is the point (0, – 4). B is the point (8, 4). C is the point (0, 12). D is the point (– 8, 4). Show that $AC = BD$, and that the angle $CAB = 45°$.

When a series of points is obtained from a known relation existing between their coördinates, it is often useful to connect the points by a line, straight or curved. This line is known as a *graph.* The following examples illustrate graphs:

PROBLEM 50. The population of a town in 1860 was 1000; in 1870, 1050; in 1880, 1120; in 1890, 1200; and in 1900, 1300. Plot a curve to show the increase in population, and from it deduce the probable population in 1885 and 1906 (Fig. 55).

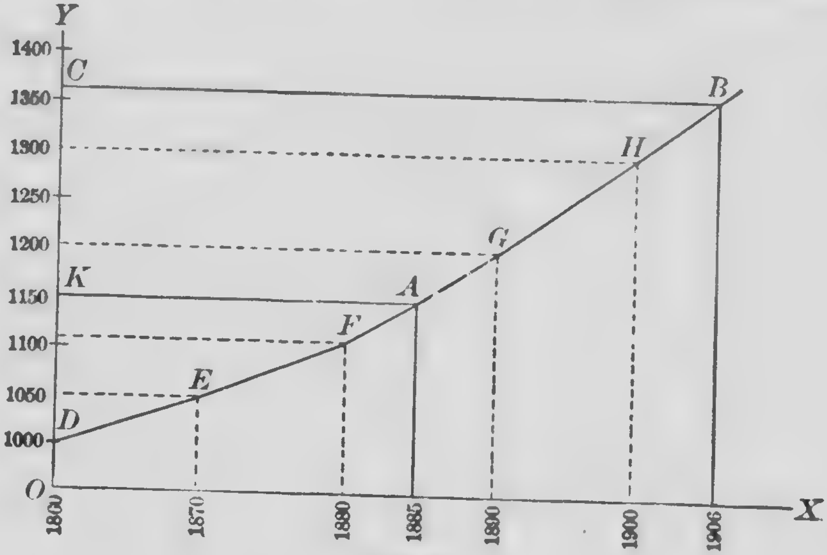


FIG. 55.

Take OX and OY for the axes. Along OX set off 4 equal distances and mark the divisions 1860, 1870, 1880, 1890, and 1900. Along OY set off a number of equal distances to represent the population at intervals of 50. Mark these divisions 1000, 1050, 1100, etc. Plot D as in the figure. Plot E directly above 1870 and abreast of 1050. Plot F directly above 1880 and abreast of 1120, and G and H in a similar way. With a freehand curve join the points *D*, *E*, *F*, *G*, and *H*. This curve, or graph, shows how the population has increased from time to time. When the increase has been comparatively slow, the curve rises slightly, as from *D* to *F*. From *F* to *H* it is steeper, and shows a more rapid increase in population.

To get the population in 1885, bisect the distance between 1880 and 1890; erect a perpendicular to meet the curve at *A*; and from *A* draw a horizontal line to meet *OY* in *K*, which is a point near the division 1150. Therefore 1150 was the population in 1885. The probable population in 1906 is got in a similar way. It is about 1370.

If we use different distances along *OX*, and also different distances along *OY*, we get a curve different from that in Fig. 55, but we obtain the same answer for the population at any given date.

PROBLEM 51. In making a tunnel, a gang of men at the end of the first day were 7 yards from the opening; and at the end of the five following days were respectively 10, 14, 12, 18, and 22 yards from the opening. Make a diagram to represent the progress of the work (Fig. 56).

Take *OX* and *OY* for the axes. Along *OX* set off equal distances to represent intervals of a day, and along *OY* set off equal distances to represent intervals of 5 yards. Determine the points *A*, *B*, *C*, *D*, *E*, and *F*, as in the accompanying figure. In this example the rate of progress depends upon the hardness of the earth met with in the excavation, and upon other causes not governed by a fixed law. Therefore it is customary to join the points by straight lines. The progress made in any particular day, compared with the progress on another day, is indicated by the inclination of these lines to OX. For example, DE is steeper than AB or EF, and therefore we infer from the diagram that the advance on the fifth day was greater than that on the second or sixth. On the fourth day an accident occurred, so that at the end of the day they were not so far advanced as at the end of the previous day. On the first day they made the greatest progress.

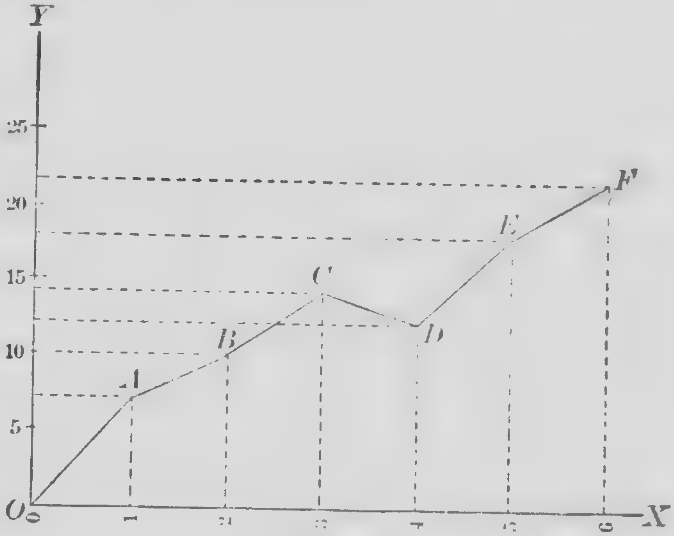


Fig. 56.

Problem 52. In 1899 the May-flower was first seen in the following counties at the following dates: Yarmouth, March 29; Shelburne, April 10; Digby, April 12; Queens, April 13; Annapolis, April 10; Lunenburg, April 12; Kings, April 14; and Hants, April 14. Plot the observations graphically (Fig. 57).

Take *OX* and *OY* as the axes. Along *OX* set off equal distances to represent the counties in the order in which they are given. Along *OY* set off equal distances to

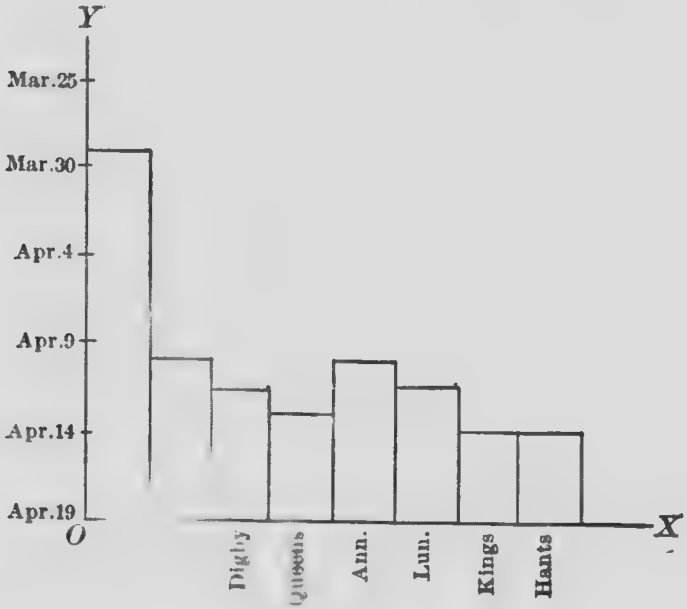


FIG. 57.

represent intervals of five days. Name the divisions and complete the diagram as in Fig. 57.

The divisions along *OY* may stand for intervals of any number of days, and the lowest division may be marked any convenient date. The record for each county is shown by the height of the horizontal line drawn across the interval for that county. This method of graphic representation is useful when the degrees, and not the rates, of variation are to be compared.

EXERCISE XXII

1. Plot the points $(0, 0)$, $(\frac{1}{4}, 1)$, $(1, 2)$, $(2\frac{1}{4}, 3)$, $(4, 4)$, and $(6\frac{1}{4}, 5)$, and connect them with a free-hand curve. When the ordinate of a point is 6, what is the abscissa? (A convenient unit is a half-inch.)

2. On a certain day the readings of a thermometer were as follows: At 8 A.M., 40°; at 9, 41°; at 10, 43°; at 11, 47°; at 12, 50°; at 1 P.M., 55°; at 2 P.M., 57°; and at 3 P.M., 56°. Draw a curve to show the variation in temperature. What was the probable temperature at 12 30? At what time was the temperature 45°? (Similar to Problem 50.)

3. Reckoned to the nearest thousand, the number of children daily present at the public schools of Nova Scotia was 50,000 in 1893, 51,000 in 1894, 54,000 in 1895, 54,000 in 1896, 55,000 in 1897, 58,000 in 1898, and 56,000 in 1899. Draw a diagram to show this graphically. (Similar to Problem 52.)

4. As recorded at Halifax, the lowest temperature in 1895 was 2° below zero; in 1896, 8° below zero; in 1897 5° below zero; in 1898, 10° below zero; in 1899, 9° below zero; and in 1900, 2° below zero. Plot the observations graphically. (Similar to Problem 52.)

5. In 1899 the May-flower was first seen in the following counties at the following dates: Guysboro, April 24; Cumberland, April 21; Colchester, April 16; Pictou, April 17; Antigonish, April 27; Richmond, April 21; Cape Breton, April 22; Inverness, April 26; and Victoria, April 27. Draw a diagram to show these observations.

6. A railroad was graded at the following rate: On Monday, 10 yards; on Tuesday, 12 yards; on Wednesday, 15 yards; on Thursday, 20 yards; on Friday, 16 yards; and on Saturday, 13 yards. Draw a diagram to show the progress from day to day. (Similar to Problem 51.)

7. Six points have the following abscissæ: 0, $\frac{1}{8}$, $\frac{1}{2}$, $1\frac{1}{8}$, 2, and $3\frac{1}{8}$; and the following ordinates: 0, $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, and $2\frac{1}{2}$ respectively. Plot the points and connect them with a curve drawn free-hand. When the ordinate of a point on the curve is 3, what is the abscissa?

8. A bridge 60 feet long is to be built across a river. Soundings at intervals of 10 feet are made to determine the depth of water, and, commencing at one bank, the depths are, respectively, 0, 3, 5, 6, $5\frac{1}{2}$, $4\frac{1}{2}$, 2, and 0 feet. Draw a curve to show the shape of the bed of the river.

9. A company has the following rates for insurance: A man of 25 years pays $21; of 30 years, $24; of 35 years, $28; of 40 years, $33; of 45 years, $39; of 50 years, $47; and of 55 years, $58. Draw a curve to show how the rates increase as a man's age increases.

10. The width of a river is 40 feet, and soundings taken at intervals of 5 feet show these depths of water: 0, 1, 7, 9, 11, 15, 13, 5, 0. Draw a section of the river bed.

CHAPTER X

THE PROJECTION OF POINTS AND LINES

HITHERTO the problems have dealt with points, lines, and figures in one plane. This chapter deals with points and lines according to their position in reference to two fixed planes at right angles to each other, and introduces the student to what is known as Descriptive Geometry. Only a few of the simplest problems on the projection of points, lines, surfaces, and solids are taken up in this book.

Descriptive Geometry is a subject on the curriculum of all science schools and colleges, and is required of those studying engineering. By means of it one is enabled to study the relations of the points, lines, and surfaces of solid bodies by reducing problems on them to problems in Plane Geometry. This will be seen by the explanations which follow.

If we place a table against the wall of a room, and on the table spread out one-half of a sheet of paper, and on the wall fasten the other half of the sheet, so that the crease in the paper follows the line of junction of the table and the wall, we have two planes at right angles to each other. A part of Fig. 58 represents this. *H.P.* is the half of the sheet on the table; *V.P.*, that on the wall; and *XY*, the crease. *H.P.* and *V.P.* are ***planes.*** *H.P.* is the *horizontal plane of projection*; *V.P.*, the *vertical plane of projection*; and *XY*, the *ground line*.

The ground line is of necessity a straight line; the other bounding lines of the planes may be straight or curved. In the diagrams, for the sake of appearance, the bounding lines are straight, either vertical or horizontal.

If we extend the horizontal plane, *H.P.*, and the vertical plane, *V.P.*, so that the one passes through the other, we have the two planes as in the diagram (Fig. 58). The observer is supposed to be standing a little to the left of the figure, so that his eyes are above the horizontal plane and in front of the vertical plane, both of which he is looking at obliquely. Lines, or parts of lines, which cannot be seen on account of the plane coming between them and the observer's eyes, are dotted lines. For instance, the observer can see the parts of the planes represented in the illustration by the sheet of paper, and therefore the bounding lines of these parts of the planes are *full* lines. When the horizontal plane is extended beyond the vertical plane, part of the bounding line cannot be seen, and hence it is shown by *dotted* lines. For a similar reason a part of the vertical plane is bounded by a dotted line.

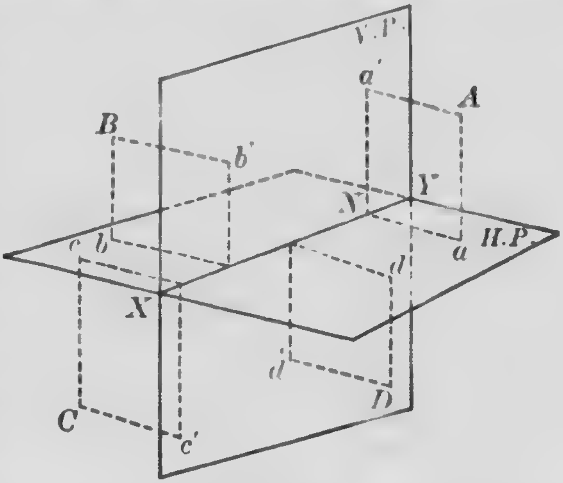


FIG. 58.

In the diagram (Fig. 58), *A* is a point in the space above the horizontal plane and in front of the vertical plane: *B*, a point above the horizontal and behind the vertical; *C*, a point below the horizontal and behind the vertical; and *D*, a point below the horizontal and in front of the vertical.

The point *A* is ***projected*** upon the two planes; that is, perpendiculars are drawn from *A* to each plane. The point where the perpendicular meets the horizontal plane is marked *a*, and *a* is called the ***horizontal projection*** or ***plan*** of *A*. Similarly, *a′* is the ***vertical projection*** or ***elevation*** of *A*. This will be clearly understood if the student returns to the illustration of the sheet of paper. When, as before, one-half is on the table and the other on the wall, place a post card in the space between the halves of the sheet so that the card stands erect on the ***H.P.*** and comes out at right angles to the ***V.P.*** Then the corner of the card on neither plane represents the point ***A***; the corner on the ***H.P.*** only, the point ***a***; and the corner on the ***V.P.*** only, the point ***a′***. Notice that the distance of ***A*** from the ***H.P.*** is measured by the length of *Aa*, and that this length is equal to ***a′N***, that is, to the distance of the vertical projection of ***A*** from the ground line. Similarly, ***Aa′*** is equal to ***aN***. The projections of ***B***, ***C***, and ***D***, also, are shown in Fig. 58.

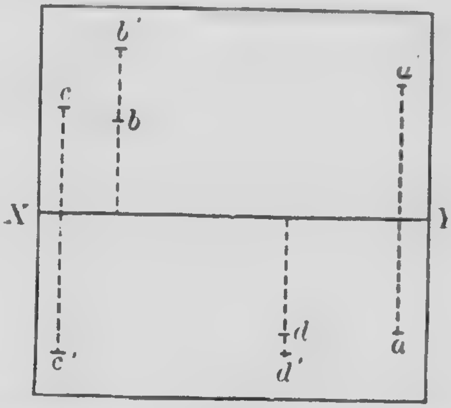
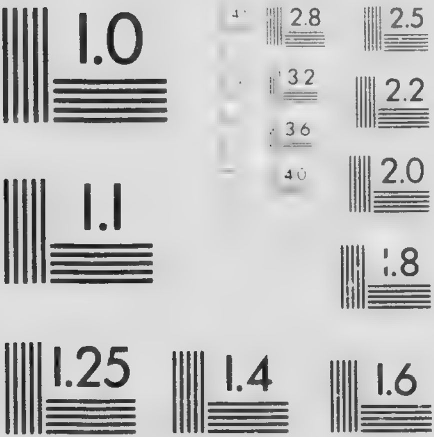


FIG. 59.

MICROCOPY RESOLUTION TEST CHART

ANSI and ISO TEST CHART No 2



APPLIED IMAGE Inc

The system of lettering should be carefully noted. Points in space are marked A, B, C, D, etc.; their horizontal projections, a, b, c, d, etc.; and their vertical projections, a', b', c', d', etc.

We have looked upon points in reference to their positions with regard to the two planes. We now consider them in respect to one plane only, but in such a way that their true relations to the two planes are still maintained. Coming again to the illustration of the paper on the table and wall, if we imagine the table drawn out from the wall and the half of the sheet formerly on the wall moved backwards until it and the other half present a flat surface of paper, the sheet of paper no longer represents two planes, but one plane only.

Referring to Fig. 58, the *V.P.* is supposed to be moved about XY as an axis till it coincides with the *H.P.*, the motion being such that the upper part of the *V.P.* recedes, and therefore the lower part moves forward. Points in the *H.P.* do not change their positions. Points in the upper half of the *V.P.* move farther away from the observer, and those in the lower half of the *V.P.* come nearer to him.

The new state of things is shown in Fig. 59. Points on the *H.P.* and in front of XY in Fig. 58 are placed below XY in Fig. 59. Hence a and d are below XY. So also are c' and d', for by the turning of the *V.P.* they come nearer to the observer. On the other hand, b and c as well as a' and b' are above XY.

When both planes are represented, the figure may be called a *pictorial* one. In it lines have not their true lengths or inclinations to one another. When only one plane is represented, the problem is resolved into one of Descriptive Geometry. Here all lines, distances, etc., are *truly* represented.

It is advisable for the beginner in every problem to draw two diagrams, the one showing points and lines with reference to the two planes of projection, the other showing their positions when the two planes are resolved into one.

To make the problems simpler and more definite, numerical values are assigned to the distances of points from the two planes, the lengths of lines, the inclination of lines to the planes, etc.

NOTE. — In the problems and exercises of this chapter the mark $''$ means inches.

PROBLEM 53. To draw the projections of the following points (Fig. 60):

1. *A* to be $1''$ above the *H.P.* and $1\frac{1}{2}''$ in front of the *V.P.*
2. *B* to be $1''$ below the *H.P.* and $1\frac{1}{2}''$ in front of the *V.P.*
3. *C* to be $\frac{1}{2}''$ above the *H.P.* and in the *V.P.*
4. *D* to be $1\frac{1}{4}''$ above the *H.P.* and $\frac{3}{4}''$ behind the *V.P.*
5. *E* to be in the *H.P.* and $1\frac{1}{2}''$ in front of the *V.P.*
6. *F* to be $1''$ below the *H.P.* and $1\frac{1}{2}''$ behind the *V.P.*
7. *G* to be in the *H.P.* and in the *V.P.*

In this problem it is not necessary to draw the diagram of the position of the points with reference to the two planes of projection, for they can be understood by reference to Fig. 58, in which the points are in some respects similarly situated.

1. *A* is situated similarly to *A* in Fig. 58. *a* is $1\frac{1}{2}''$ from *XY*, and in front of it. *a′* is above *XY*, and when the *V.P.* is turned *a′* is on the other side of *XY* and $1''$ from it. Also *a′* is directly opposite to *a*. Hence the following construction: Draw a ground line *XY*, and the line *a′a* at right angles to it. Measure the part above *XY* $1''$ in length and the part below $1\frac{1}{2}''$ in length. Thus the vertical and horizontal projections *a* and *a′* are determined.

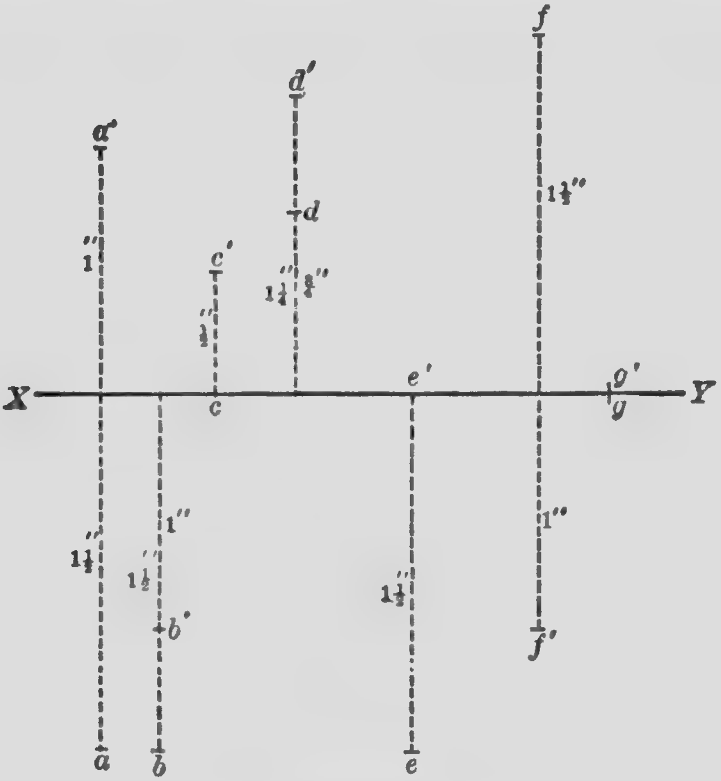


FIG. 60.

2. B is similar to D in Fig. 58. Its horizontal projection is farther from the ground line than its vertical projection is. Hence, when the *V.P.* is turned, b' comes directly between b and the ground line. Hence the following construction: Draw a line below XY and perpendicular to it. Measure off the lengths $1\frac{1}{2}''$ and $1''$. This determines the points b and b'.

3. When C is in the *V.P.*, its horizontal projection is in the ground line. When the *V.P.* is turned, c' recedes. We therefore draw a line above XY $\frac{1}{2}''$ long and perpendicular to it, and get the points c and c'.

4. D is similar to B in Fig. 58. When the *V.P.* is turned, both projections are behind the ground line, the vertical projection being farther from the ground line than the horizontal one. Hence draw a line at right angles to XY and above it, and measure lengths $\frac{3}{4}''$ and $1\frac{1}{4}''$. Thus the points d and d' are determined.

5. E being in the *H.P.*, its vertical projection is in the ground line. Its horizontal projection is $1\frac{1}{2}''$ in front of the ground line. Draw a line $1\frac{1}{2}''$ long below XY and perpendicular to it, getting the points e and e'.

6. F is similar to C in Fig. 58. When the *V.P.* is turned, c' is raised and comes in front of the ground line. c remains behind the ground line. We therefore draw a line at right angles to XY, and make the part above XY $1\frac{1}{2}''$ long and that below $1''$ long, thereby getting the points f and f'.

7. For G to be in the *H.P.* and the *V.P.* it must be in the ground line, the only line common to the two planes. Its two projections coincide. They are shown by the point g, g'.

Problem 54. *A* is a point $\frac{3}{4}''$ above the *H.P.* and $1''$ in front of the *V.P.* To find the distance of *A* from the ground line (Figs. 61, 62).

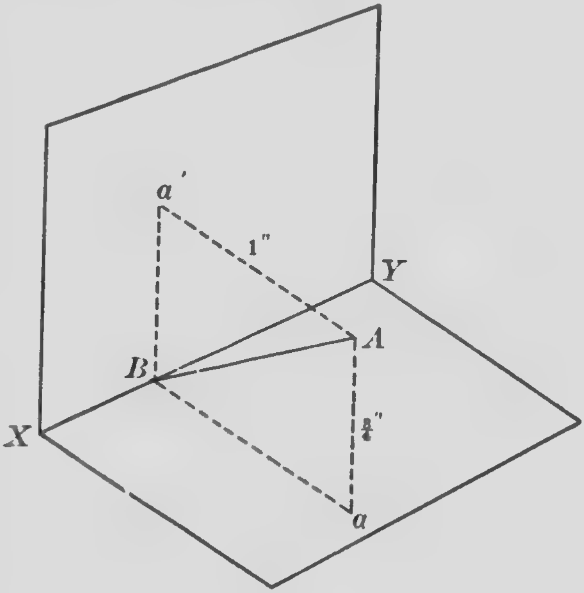


Fig. 61.

Take *B*, any point in the ground line, and erect a perpendicular *Ba′*. This line will be in the *V.P.* and its length will be called $\frac{3}{4}''$. Also from *B* draw *Ba*, a perpendicular to the ground line, $1''$ long and wholly in the *H.P.* Complete the rectangle *aBa′A*. Thus *A* is found.

AB is the distance of *A* from the ground line. *AaB* stands for a right-angled triangle, of which *Aa* and *Ba* are the two perpendicular sides. Find, as in Fig. 62, *a* and *a′*, the projections of *A*. With *B* as a centre and *Ba* as radius, describe the arc *aC*, meeting *XY* in *C*. *a′C* is the required length. Its

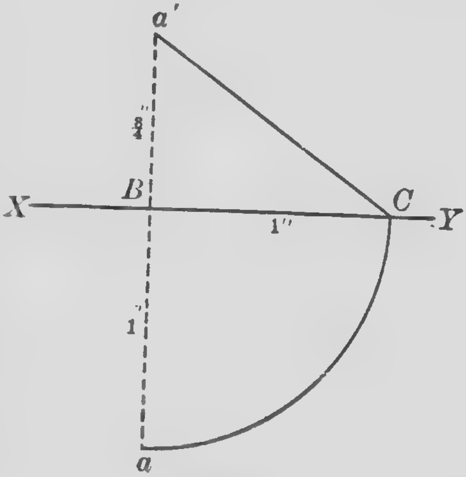


Fig. 62.

length as found by the scale is $1\frac{1}{4}''$. Its length may be got by calculation, for the square on $a'C$ is equal to the sum of the squares on $a'B$ and BC, that is, is equal to $\frac{9}{16}+1=\frac{25}{16}$; $\therefore a'C=\sqrt{\frac{25}{16}}=\frac{5}{4}$.

PROBLEM 55. To project the following lines:

1. *AB*, $1\frac{1}{2}''$ long, $\frac{3}{4}''$ in front of the *V.P.*, and standing erect on the *H.P.*

2. *CD*, $1\frac{1}{4}''$ long, $\frac{1}{2}''$ above the *H.P.*, and standing out directly from the *V.P.*

3. *EF*, $1''$ long, $1''$ above the *H.P.*, $\frac{1}{2}''$ in front of the *V.P.*, and parallel to both the *H.P.* and *V.P.* (Figs. 63, 64).

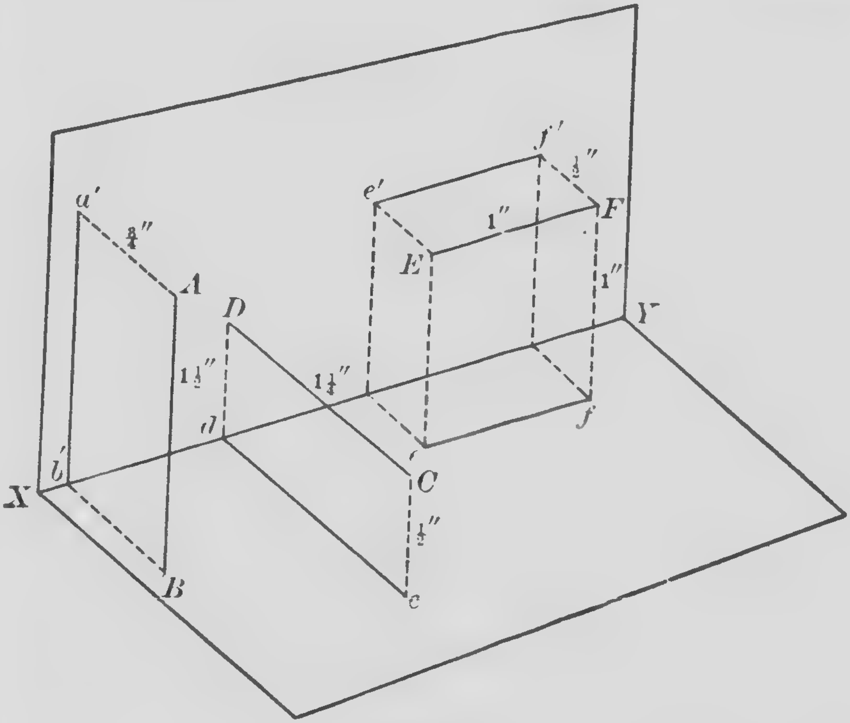


FIG. 63.

1. The horizontal projections of *A* and *B* coincide, and in Fig. 64 are marked *a b* and are $\frac{3}{4}''$ in front of the *V.P.* in Fig. 63, and therefore $\frac{3}{4}''$ below *XY* in Fig. 64. The horizontal projection of the line *AB* is a point, and the vertical projection is *a′b′*, $1\frac{1}{2}''$ long, and above the *H.P.* in Fig. 63, and therefore above *XY* in Fig. 64.

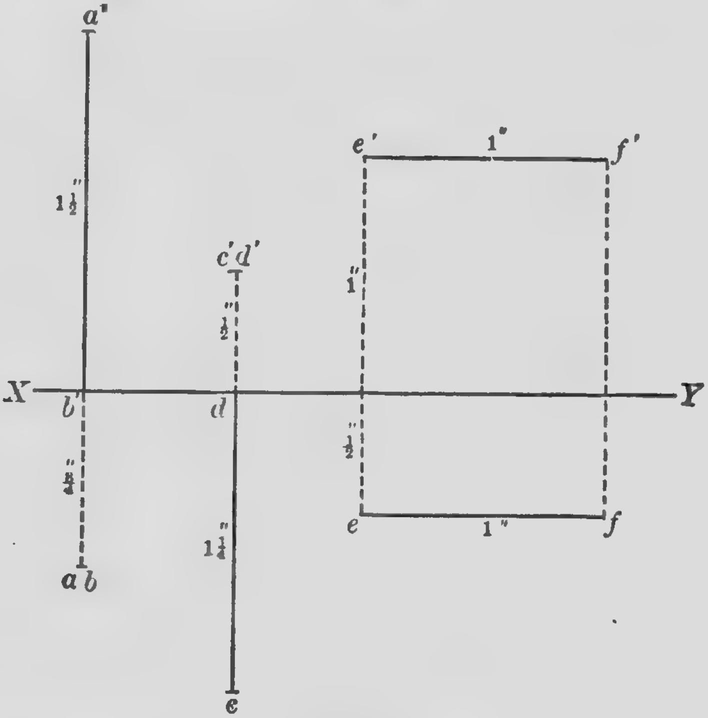


FIG. 64.

2. The horizontal projection of *CD* is $1\frac{1}{4}''$ long and is in front of the *V.P.* in Fig. 63, and therefore below *XY* in Fig. 64.

3. Since *EF* is parallel to the *H.P.* and to the *V.P.*, its projections are parallel to the ground line. Its horizontal projection is $1''$ long, and is $\frac{1}{2}''$ in front of the

V.P. in Fig. 63, and therefore $\frac{1}{2}''$ below *XY* in Fig. 64. Its vertical projection is 1″ long, and is 1″ above the *H.P.* in Fig. 63, and therefore 1″ above *XY* in Fig. 64.

PROBLEM 56. To draw the projections of a line *AB*, 1″ long, in the following positions:

1. Lying in the *H.P.*, at an angle of 60° to the *V.P.*, and one end touching the *V.P.*
2. Parallel to the *H.P.*, at an angle of 60° to the *V.P.*, $\frac{1}{2}''$ above the *H.P.*, and one end touching the *V.P.*
3. Lying in the *V.P.*, at an angle of 60° to the *H.P.*, and one end touching the *H.P.*
4. Parallel to the *V.P.*, at an angle of 60° to the *H.P.*, $\frac{1}{2}''$ in front of the *V.P.*, and one end touching the *H.P.* (Figs. 65, 66).

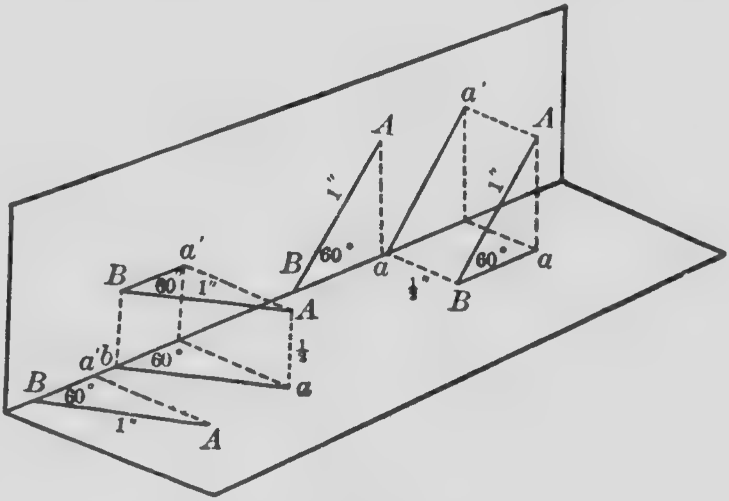


FIG. 65.

1. As *AB* is in the *H.P.*, its vertical projection lies in the ground line. The line itself is the horizontal projection, and *a′B* is the vertical projection.

2. As *AB* is parallel to the *H.P.*, its horizontal projection is 1″ long and makes an angle of 60° with the ground line: and its vertical projection is parallel to the ground line and ½″ above it.

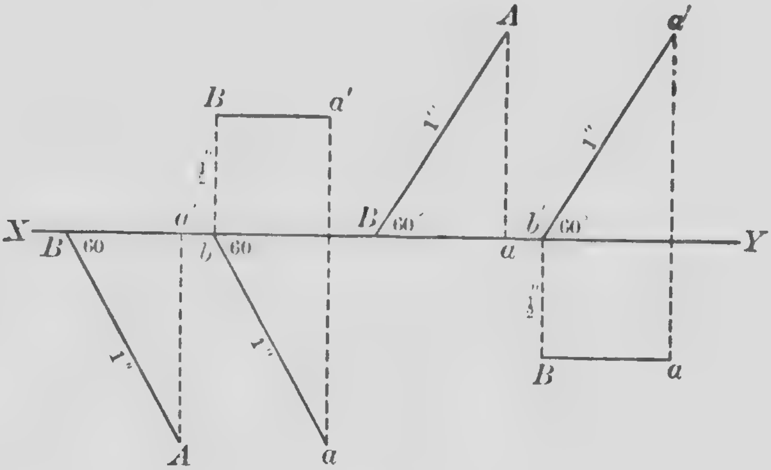


FIG. 66.

3. As *AB* is in the *V.P.*, its horizontal projection lies in the ground line. The line itself is the vertical projection and *aB* is the horizontal projection.

4. As *AB* is parallel to the *V.P.*, its vertical projection is 1″ long and makes an angle of 60° with the ground line; and its horizontal projection is parallel to the ground line.

EXERCISE XXIII

NOTE. — In each of the following exercises two diagrams should be drawn, the one showing both planes, and the other one plane only.

1. *A* is 1¼″ above the *H.P.* and ¾″ in front of the *V.P.*; *B* is 1½″ above the *H.P.* and ½″ in front of the *V.P.*; *C* is 1″ above the *H.P.* and 1″ behind the *V.P.*;

and *D* is $\frac{3}{4}''$ below the *H.P.* and $1\frac{1}{4}''$ behind the *V.P.* Represent these points and their projections.

2. Find the distance of each point in No. 1 from the ground line.

3. Draw the projections of *AB*, $1\frac{1}{2}''$ long, at right angles to the *V.P.* and $\frac{3}{4}''$ above the *H.P.*, its back end *B* being $\frac{1}{2}''$ in front of the *V.P.*

4. Of the same line, at right angles to the *H.P.*, $\frac{3}{4}''$ in front of the *V.P.*, its lower end *B* being $\frac{1}{2}''$ above the *H.P.*

5. Of the same line lying in the *H.P.* and making an angle of 30° with the ground line, its back end *B* being $\frac{3}{4}''$ in front of the *V.P.*

6. Of the same line, parallel to both planes and $1''$ from each.

7. Represent a ladder with its top reaching a window 15 feet from the ground and with its base 20 feet from the wall. Use the scale of 10 feet to an inch and neglect the breadth of the ladder. Draw its projections.

8. Represent a needle, 2 inches long, stuck in a wall so as to make an angle of 45° with it and to be parallel to the floor and 1 inch from it. Draw its projections.

9. Place a table against the wall. Put a post card on the table. With a short edge touching the wall and the right-hand edge remaining on the table, lift the other edge till the card turns through angles of 30°, 45°, 60°, and 90°. Draw a diagram to show the projections of the card in these four positions. Scale of 2 inches to an inch.

10. Keeping the short edge of the card in No. 9 touching both the table and the wall, turn the card about that edge through angles of 30°, 45°, 60°, and 90°. Draw a diagram to show the projections of the card in the four positions.

CHAPTER XI

WORKING DRAWINGS

BY *working drawings* are meant those drawings so made with mathematical precision that any one constructing an object from them may take any measurement or shape required directly from the drawing, by applying the scale, compasses, or other instrument.

A little study of Fig. 67 will show that a *pictorial* or *perspective* drawing of an object will not serve for a working drawing. The object represented is a box. The angles at the corners of the box are really right angles, and the lines *AB*, *DC*, and *FG* are really equal to one another. But in the diagram, *DC* is longer than either *AB* or *FG*, and *DAB* is an angle less than 90°, while *ADC* is an angle greater than 90°. It requires a great deal of mathematics to determine the real lengths of lines and exact sizes of angles from a *true* perspective drawing. Such a drawing, therefore, is not suitable for a working drawing.

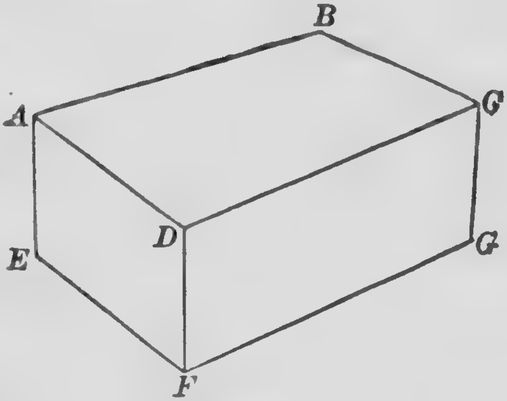


FIG. 67.

In order that any dimension may be taken directly from the drawing, a *projection* drawing, or *orthographic* projection, is made.

In the perspective drawing (Fig. 67), three sides of the box are shown. In a projection drawing, however three diagrams are needed to show the three sides.

Figures 63–66 of Chapter X illustrate the method of projecting lines upon the planes of projection. Similarly

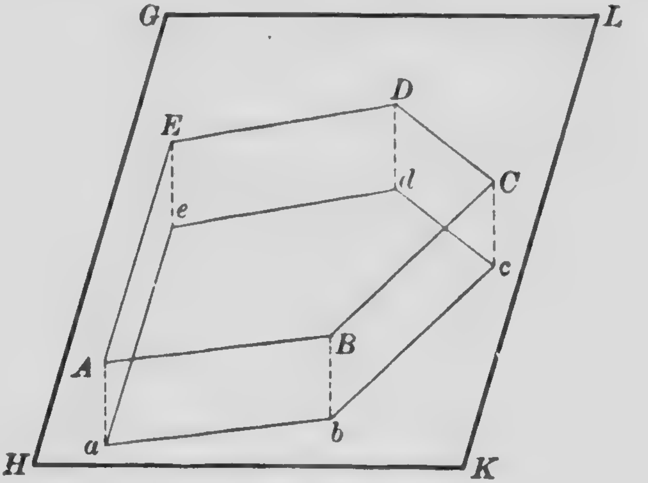


FIG. 68.

any surface may be projected upon a plane. Figures 68 and 69 represent this. *GHKL* is the plane on which the surface ***ABCDE*** is to be projected. To find its projection proceed thus: From *A*, *B*, *C*, *D*, and *E*, drop perpendiculars to the plane, intersecting it in the points *a*, *b*, *c*, *d*, and *e*. Join these points in order. Then the figure *abcde* is the projection of the given surface. In Fig. 68 the surface is regarded as being parallel to the plane *GHKL*. Then, *A*, *B*, *C*, *D*, and *E* are equidistant from the plane, and, therefore, the perpendiculars *Aa*, *Bb*, etc., are equal to one another. It follows that $AB = ab$, $BC = bc$, etc., and that the projection *abcde* is equal to the surface *ABCDE*.

In Fig. 69 the surface is regarded as being oblique to the plane *GHKL*. In this case, the points *A*, *B*, *C*, *D*, and *E* are not equidistant from the plane, and, therefore, the perpendiculars *Aa*, *Bb*, etc., are not equal to one another. For example, *C* is farther from the plane than *B* is. Hence *Cc* is greater than *Bb*, and therefore *BC* is greater than *bc*. It follows that the projection *abcde* is smaller than the surface *ABCDE*.

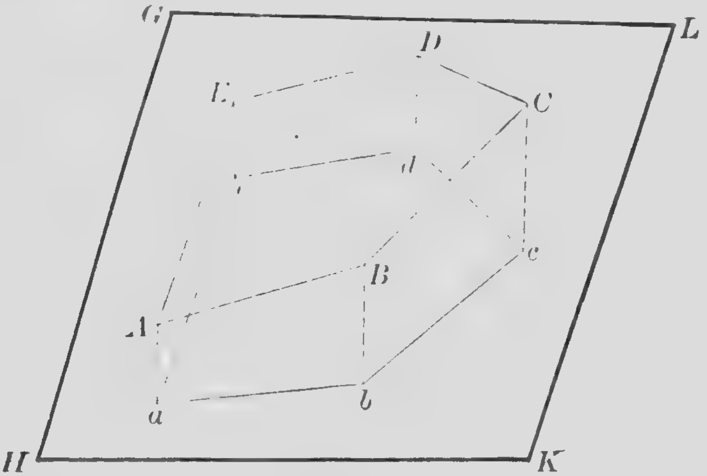


FIG. 69.

In the simplest drawings, the object to be represented is assumed to be so placed that one of its flat surfaces, or one of its centre lines, is parallel to one of the planes of projection. This method can be adopted when single pieces of a complex object are represented separately.

In representing complicated pieces where the parts are joined at various angles, such an assumption cannot conveniently be made. For such pieces of work many different drawings may be needed. Each piece that cannot be shown in true shape and proportion upon the plan and elevation of the whole must be given in a plan and elevation separately. For example, a joining not shown in the general view must be shown by giving the plan or elevation, or both, of a cutting through the joining.

Before giving any working drawings, it will be well to notice the character of the lines used in drawings which are made in one color only:

1. The full line ____________, used in outlining figures and in representing lines of the object that can be seen.

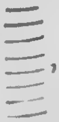
2. The dotted line ------------------, made up of short dashes, to represent a line of the object which is hidden in the view shown.

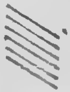
3. The broken and dotted line — - — - — - —, to represent an axis of the object, and also when a sectional view is given, to show where the section has been made.

4. The broken line of long dashes,

$$A \longleftarrow 1\tfrac{3}{10}'' \longrightarrow| B,$$

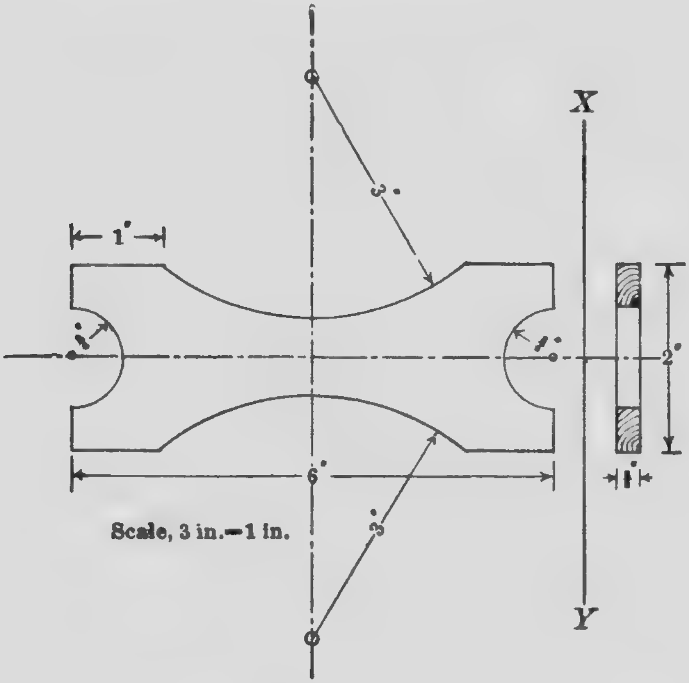
used for dimension lines. This means that the distance from A to B is $1\frac{3}{10}$ inches.

5. The freehand curves , to show a view across the fibre of the wood.

6. The parallel lines , oblique to the axis, to indicate views of sections.

The student should now be able to *read* a simple working drawing, to understand the diagrams that follow, and to make for himself the drawings asked for in Exercise XXIV.

PROBLEM 57. To make a working drawing of a fishing-line winder to be constructed from a piece of wood, 6 inches long, 2 inches wide, and ⅜ inch thick (Fig. 70).



Scale, 3 in.=1 in.

FIG. 70.

To return to the illustration of the table against the wall (see page 77). Regard the winder as being parallel to the table, at right angles to the wall, and equidistant from both. Then the projection on the table, or the *plan*, is equal to the upper surface of the winder ; and the projection on the wall, or the *elevation*, is equal to one of the ends. In the diagram, the plan is to the left of ***XY***, and the elevation to the right. The curves at the ends of the winder are semicircles of ¾ inch radius, and the curves at the sides are arcs of 3 inches radius. The length of the winder is marked on the plan, and the width and thickness are marked on the elevation. The grain of the wood is shown by the freehand curves in the elevation.

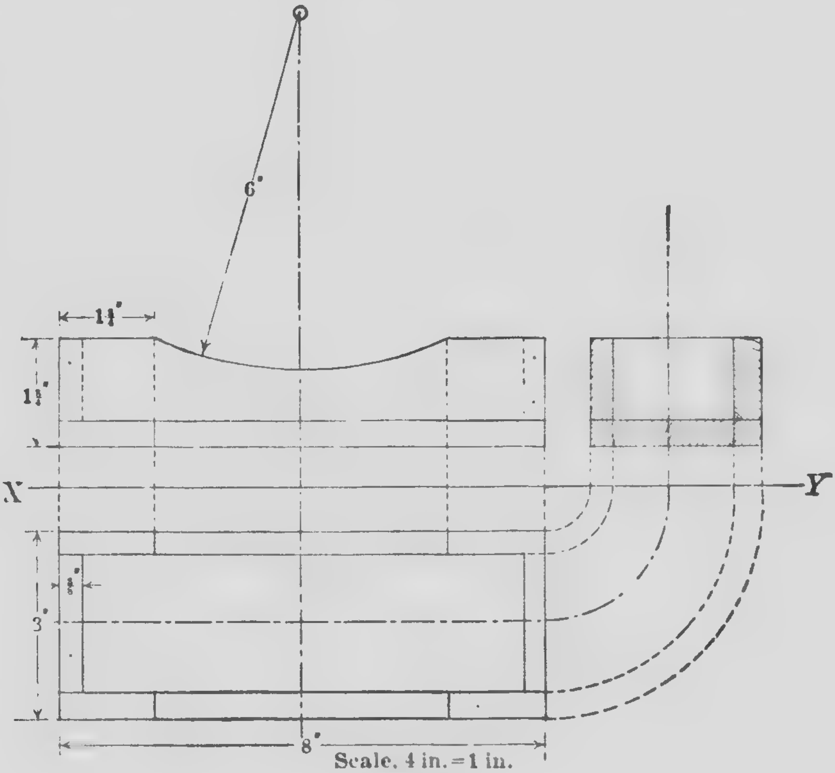


FIG. 71.

PROBLEM 58. To make a working drawing of a pencil tray, 8 inches long, 3 inches wide, 1¾ inches deep, and made of wood ⅜ inch thick (Fig. 71).

Regard the tray as being placed in a position similar to that of the winder. Then the plan of the tray is equal to its bottom surface, and the two levations to a side and an end respectively. In the diagram, the plan is below *XY*, and the elevations are above. The three series of curves in the elevation of an end indicate that the parts so represented are three distinct pieces. The parts of the tray are to be held together by twelve nails placed in a regular order. Their positions are shown by the dots in

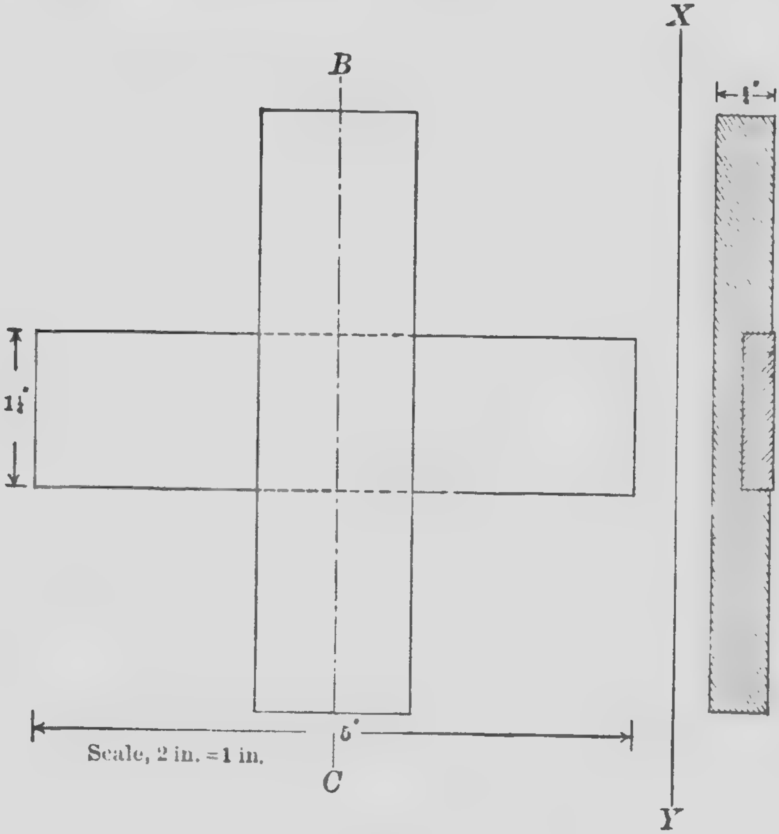


FIG. 72.

the diagram. The thickness of the wood and dimensions of the tray are marked on the diagram.

PROBLEM 59. To make a working drawing of a cross-halving joint. The wood is $\frac{3}{4}$ inch thick, and the other dimensions are as marked on Fig. 72.

The cross is supposed to be placed parallel to the *H.P.*, and to have its sectional line *BC* parallel to the *V.P.* A vertical cutting is made along *BC*, and its elevation is shown to the right of *XY*. The view of the section is indicated by parallel lines, of which there are two sets to show that this section is made up of two distinct pieces.

EXERCISE XXIV

1. Make a drawing of Fig. 70, true size.
2. Make a drawing of Fig. 71, true size.
3. Make a drawing of Fig. 72, true size.
4. Draw the plan and elevation of a calendar-back, here represented in perspective. The particulars are: greatest length, 6 inches; breadth, $4\frac{1}{4}$ inches; thickness, $\frac{3}{8}$ inch; radius of curve, 3 inches; centre of curve, equidistant from the sides; at the middle points of the first, third, and fifth radii, holes $\frac{3}{8}$ inch in radius are to be bored, and at points $\frac{1}{2}$ inch from the middle points, holes $\frac{1}{4}$ inch in radius are to be bored; and on

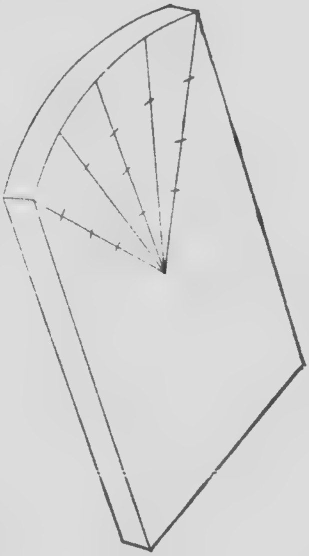


FIG. 73.

the second and fourth radii, holes $\frac{1}{4}$ inch in radius are to be bored.

5. Draw a plan and an elevation of a rectangular photograph frame to be made according to the following directions: length, 7 inches; breadth, 6 inches; thickness, $\frac{3}{8}$ inch; the opening, rectangular and $1\frac{1}{4}$ inches from the edges of the frame; the strips at the back, three in number, each $\frac{1}{2}$ inch wide and $\frac{1}{4}$ deep, and nailed at a distance of $\frac{1}{2}$ inch from the edge of the opening; the back, just covering the strips and $\frac{1}{8}$ inch thick.

6. Draw a side elevation and a plan of a frame through which boys roll marbles. The frame is 12 inches long, 3 inches wide, $\frac{3}{4}$ inch thick, and has five openings at regular intervals. The length of each of the parallel sides of the openings is 1 inch, and the tops of the openings are semicircles whose radii beginning at an end are respectively $\frac{3}{4}$, $\frac{7}{8}$, 1, $\frac{7}{8}$, and $\frac{3}{4}$ inches.

7. A pin tray is to be made from a block of wood 6 inches long, 2 inches wide, and 1 inch deep. The groove is to be $\frac{1}{4}$ inch from the sides, to have semicircular ends, and to be gouged to a depth of 1 inch. Draw a plan and two sections, the one section vertical and through the middle of the tray lengthwise, the other section vertical and through the middle of the tray crosswise.

8. Draw a plan, an elevation, and a cross section of a trunk. Length is 9 inches; breadth, 3 inches; thickness of main parts, $\frac{3}{8}$ inch; inside depth, 2 inches, and lining of the cover, $\frac{1}{4}$ inch thick. The trunk is so made that an end of each of the sides, the end of the bottom, and the end of the cover are in full view.

9. Make a working drawing of a wall bracket, giving

a plan and a front and a side elevation. Do your own designing, taking into account that the three pieces of the bracket are to be made out of one piece of material, which is $14 \times 4\frac{1}{4} \times \frac{1}{2}$ inches.

10. Design and give a working drawing of a test-tube holder.

MISCELLANEOUS EXERCISES

NOTE. Exercises taken from the examination papers for the High Schools of Nova Scotia are indicated by the letter *C* or *D*.

1. Draw a quadrilateral whose sides shall be 36, 48, 54, and 42 feet, and the diagonal joining the ends of the first and second, 60 feet. Measure the length of the other diagonal, the size of each of the four angles of the quadrilateral, and its area.

2. Draw a square each side of which is 1 dm. long. On one side of the square draw a triangle whose two remaining sides shall be respectively .68 and .76 of a decimeter and within the square. Join the apex of the triangle with the remaining angles of the square. Find by measurement the lengths of these two sides and the degrees in each of the four angles at the apex of the original triangle. (*C*, 1895.)

3. On a line BC 120 inches in length describe a triangle ABC, having the angle at B 44°, and that at C 62°. Bisect each of these angles by lines meeting at D. Measure the lengths of AB, AC, DA, DB, and DC.

4. A ladder is 30 feet long. When its foot is in a certain place on the ground its top will reach a window on one side of the street 20 feet from the ground, or a window on the other side of the street 25 feet from the

ground. Draw a diagram to represent this and find the width of the street.

5. On a line *BC*, 48 feet long, describe first a triangle *ABC* having the angles at *B* and *C* 48° and 54° respectively; and secondly, a square *BDEC* on the other side of *BC*. Find the distance of *A* from each corner of the square.

6. On the scale of 100 feet to an inch draw the plan of a plot of land, *ABCDE*, bounded as follows: From *A* to *B* 456 feet, thence 80° to the right 316 feet to *C*, thence 50° to the right 278 feet to *D*, thence 65° to the right 509 feet to *E*, thence to the point of commencement at *A*. Find the lengths of *AE*, *AD*, and *AC*, and the internal angles at *A* and *E*; and, by drawing perpendiculars from the vertex to the base of each triangle, find the area in square feet of the whole plot. (*C*, 1898.)

NOTE. See note under exercise 20, page 44.

7. Draw a rhombus 20 feet on a side, the length of the longer diagonal being 32 feet. Find the length of the other diagonal, and the area of the rhombus.

8. To measure the height of a flagstaff standing on the top of a building a man proceeded thus: At a certain point on the ground he measured the angle of elevation of the top of the staff and found it to be 47°. He then walked 100 feet directly towards the building, and at the point reached measured the angles of elevation of the tops of the staff and building, and found them to be 82° and 71° respectively. How long was the staff?

9. Draw a regular pentagon on a base of 40 feet.

Find its centre, the distance of the centre from a side, and the area of the pentagon.

10. The first, second, and third sides of a quadrilateral are respectively, 95, 285, and 120 feet. The angle contained by the first and second sides is 120°, and that by the second and third sides is 65°. Find the fourth side and the other angles. (*D*, 1899.)

11. A tower is in a level country. To determine its height a man measured its angle of elevation as observed from a certain point on the ground and found it to be 43°; and that as observed at a point 200 feet nearer to the tower he found to be 70°. How high was the tower?

12. Draw the hexagon $ABCDEF$ and measure its angles, given $AB = 69$, $BC = 87$, $CD = 77$, $DE = 51$, $EF = 67$, $FA = 63$, $AE = 121$, $AD = 163$, and $BD = 149$.

13. A telephone post is 60 feet high and an electric light post is 35 feet high. The distance between them is 30 feet. How far from the shorter post must a man whose eye is 5 feet from the ground stand in order to just see the top of each post?

14. A ship sails east 20 miles, northwest 45 miles, and northeast 60 miles. Find the distance and direction from the point of starting. (*C*, 1899.)

15. Three lines OA, OB, and OC are drawn from O, making equal angles with each other. $OA = 215$ inches, $OB = 259$ inches, $OC = 307$ inches. Find the lengths of AB, BC, and CA. (*C*, 1903.)

16. $ABCD$ is a quadrilateral. $AB = 40$, $AD = 25$, and $AC = 35$ inches. Also angle $BAC = 27°$, and angle $CAD = 43°$. Draw the quadrilateral and measure BC, BD, and CD. Also find the area of the quadrilateral.

17. Draw the pentagon *ABCDE*, given *AB* = 320, *BC* = 240, *CD* = 400, *DE* = 360, and *EA* = 200 feet; and angle at *B* = 110°, and angle at *C* = 117°. Measure *AC* and *BD*.

18. A man standing at a distance from a cathedral finds that the top of the spire is elevated 30° above the horizontal line on a level with the base of the cathedral; he then walks 200 feet directly towards the cathedral on that line and finds the top of the spire to be elevated 60°; find, by plotting, the height of the spire. (*C*, 1900.)

19. If you leave a certain point and walk 40 feet to the east, then 60 feet to the northeast, then 70 feet to the north, then 30 feet to the northwest, and then 100 feet to the southwest, how far is the point at last reached from the starting-point and from each of the angular points?

20. From a point *O* within a pentagon the distances to the corners *A*, *B*, *C*, *D*, and *E* are respectively 36, 29, 32, 30, and 35 feet, and the angles *AOB*, *BOC*, *COD*, and *DOE* are respectively 71°, 56°, 49°, and 81°. Draw the pentagon and find the length of each side.

21. A mine has a vertical shaft 200 feet deep. From the bottom a tunnel is made for a distance of 300 feet on an upward slope of 20°. It is then continued at a slope of 30°. How far from the mouth of the shaft will the tunnel strike level ground?

22. By plotting find approximately the parts and area of the triangle *BCE* within the quadrilateral *ABCD*; given *AD* = 147, *DC* = 235, angle *ADC* = 90°, angle *DCB* = 95°, angle *BAD* = 111°, *AE* = 95, and *DE* = 95. (*C*, 1901.)

23. A man stands 20 feet from a lamp-post. If his shadow is 4 feet long and he is 6 feet tall, how high is the lamp?

24. Two straight lines *AB* and *CD* are each 456 inches long; they cut each other at right angles in the point *O*, so that *AO* is 115 inches long and *CO* 258 inches. Find the length of *CB* and the number of degrees in the angle *CBD*.

25. Make a plan of a field *ABCDEF* from these notes: I walk from *A* due east to *D* 200 yards; when I am at *G*, 50 yards from *A*, *B* is due north of me and *F* is due south, each 60 yards; and when I am at *H*, 70 yards from *D*, *C* is due north of me and *E* is due south, each 80 yards. Find the area of the field and the distance round it.

26. Draw the pentagon *ABCDE*, having $AB = 30$, $BC = 36$, $CD = 40$, $DE = 32$, $EA = 32$, angle at $B = 100°$, and angle at $C = 94°$. Measure the other angles and find the area of the pentagon. (The figure has no reëntrant angles.)

27. A man in a boat observes the angle of elevation of a cliff to be 40°, and that of a tower 50 feet high standing on the cliff to be 50°. How high is the cliff and how far from its base is the boat?

28. A beacon stands on the summit of a hill. At a point in the plain the angle of elevation of the beacon is 20°, and at a point 70 yards nearer the hill it is 35°. Draw a diagram, and find the height of the beacon above the level of the plain.

29. A schooner wishes to sail due north, but meets with a head wind. It therefore sails 10 miles northeast, then 16 miles west, then northeast; and now finds itself

due north of its starting-point. How many miles has it lost by being obliged to tack?

30. *CD* is a tower standing on the inclined plane *AC*; *AB* is a base line in this plane; at *A* the angle of elevation of the top of the tower is 45°, and at *B* it is 63°. *AB* is 112 feet long and inclined 20° to the horizon. Find the height of the tower.

31. A man measures *AB*, one side of a triangular field, and finds it to be 200 meters; he measures the angle at *B* and finds it to be 72°; he walks from *B* to *C*, but neglects to measure *BC*; he finds the angle at *C* to be 46°. Draw a plan of the field and find how long *BC* is.

32. Three vessels left the same port and sailed in different directions. How far apart will they be after the first sails 10 kilometers to the south, the second 14 kilometers to the southwest, and the third 18 kilometers to the northeast?

33. The angle of elevation of the top of a tower to a person standing on the opposite bank of a river is 60°. When he has retired 100 feet in a straight line from the river, the angle of elevation is 45°. Find the breadth of the river and the height of the tower.

34. A ladder placed at an angle of 75° with the ground just reaches the sill of a window 27 feet above the ground on one side of the street. On turning the ladder over without moving its foot, it is found that when it rests against a wall on the other side of the street it is at an angle of 15° with the ground. Find the breadth of the street.

35. *ABCDE* is a field having the following dimensions: *AB* 200 yards, *BC* 240 yards, *CD* 250 yards, *DE* 300

yards, *EA* 230 yards, *AC* 350 yards, and *AD* 360 yards. Draw a diagram of the field on a scale whose representative fraction is $\frac{1}{3000}$, and find the area.

36. A telescope is mounted so that it has a range of the sea from the east to the north. A ship sailing in a northwest direction appears in view and continues on the same course. Her nearest distance away was 5 miles. For how many miles did she sail within view?

37. Describe regular pentagons on bases of 3 and $1\frac{1}{2}$ inches long respectively and compare their areas.

38. I observe the angle of elevation of the top of a tree to be 15°, and that of a taller tree behind it to be 30°. I walk 30 yards towards them, and at the point reached notice that their tops are in a straight line at a common elevation of 45°. Find the heights of the trees and the distance between them.

39. A buoy is moored 400 yards from a shore from which the land has an upward slope, first of 20° to the horizon for 500 yards, and then of 35° for 400 yards. How high above the buoy is the top of the slope? Express the answer in (1) degrees and (2) yards.

40. *A*, *B*, and *C* are three towns in a level country. *A* is 2 miles west and 5 miles north of *B*. *C* is 5 miles west and 8 miles south of *B*. Plot these points. Measure the distances between the towns. What direction is *C* from *A*?

ANSWERS

PART I

Exercise I

1. 1.86 in., 1.08 in. **7.** 1.26 in.

Exercise II

3. 6.09 in., $D = 34°$, $E = 36°$. **9.** 77°, 103°.

Exercise III

1. 121°, 43°. **4.** $A = 56°$, $B = 51°$, $C = 73°$.

7. $AD = 3.77$ in., $BD = 2.82$ in., $A = 45°$, $B = 71°$, $D = 64°$.

8. 30°, 90°, 15°, 120°, 60°. **9.** $3\frac{1}{3}$ in. **10.** 66°, 114°.

Exercise IV

3. 100 ft., 207 ft., 50 ft. **4.** 264 ft. **6.** 6.71 mi.

7. 20 ft. **8.** 5.14 mi. **9.** 49.5 ft., 51.3 ft., 76°, 69°.

10. 6.4 mi., 7.81 mi., 8.94 mi., 10 mi.

Exercise V

7. 9.57 in. **9.** $AD = 3.43$ in., $DB = 2.05$ in., $A = 36°$, $B = 84°$.

10. $BC = 2.9$ in., $DE = 1.45$ in.

Exercise VI

5. 13 mi. **10.** 5.27 in., 5.46 in., 3.76 in., 3.9 in.

Exercise VII

7. $AD = BC = 2.6$ in. **8.** 1.3 in. **9.** 1128 ft.

Exercise VIII

5. Each perpendicular = 1.47 in. **9.** 32.7 mi. **10.** 17.7 mi.

Exercise IX

1. 2.06 in. **3.** 1.08 in., 1.94 in. **4.** 1.8 in., 1.39 in., 1.55 in.

5. $A = 64°$, $B = 55°$, $C = 61°$. **6.** 1.56 in., 1.26 in. **7.** 49°, 59°, 72°.

8. 53°, 37°, 90°. **9.** 23°, 67°, 90°. **10.** 675 ft., 24°, 19°.

Exercise X

1. 1.73 in., 58°, 69°; 1.84 in., 2.02 in., 53°.
2. 1.94 in., 1.54 in., 63°; 1.73 in., 1.93 in., 53°, 64°.
4. 1.9 in., 1.49 in., each side = 1.77 in.; angles are 57°, 57°, 66°.
6. 113°, 1.64 in. **9.** 1.55 in.
10. $DA = 3.46$ in., $DB = 2.21$ in., $A = 38°$, $B = 76°$, $D = 66°$.

Exercise XI

3. Each segment = 1.63 in.
10. $EC = 1.13$ in., $ED = 2.71$ in., $BEA = 92°$, $BEC = 114°$, $CED = 80°$, $DEA = 74°$.

Exercise XII

1. 1.85 in., 90°, 45°. **2.** 1.81 in., 67°, 113°, $56\frac{1}{2}°$.
3. 2.57 in., 1.17 in., 90°, 25°. **4.** 2.24 in., 1.55 in., 50°, 130°.
7. 1.93 in. **8.** 2.84 in., 1.91 in. **10.** 1.25 dm., .62 dm.

Exercise XIII

1. $A = 106°$, $C = 96°$, $D = 57°$. **2.** $CD = 1.65$, $C = 83°$, $D = 77°$.
3. $AD = 2.91$ in., $DC = 2.24$ in.
5. $A = 118°$, $B = 91°$, $C = 89°$, $D = 62°$.
6. $BC = 4.1$ cm., $CD = 3.67$ cm. **7.** $B = 131°$, $C = 62°$, $D = 131°$.
8. 4.07 in. **9.** $DC = 2.06$ in., $EF = 3.03$ in. **10.** 1.71 in.

Exercise XIV

2. 10.8 sq. in. **3.** 4.56 sq. in. **5.** 15.9 sq. in. **7.** $11\frac{1}{4}$ sq. in.
8. $8\frac{5}{16}$ sq. in.
10. (1) 800 sq. ft., (2) 1250 sq. ft., (3) 940 sq. ft., 5980 sq. ft.

Miscellaneous Exercises

1. $BC = 1.5$ in., $CD = 2$ in., $ABC = 110°$, $ADC = 110°$, $BCD = 70°$.
3. 37°, 53°, 90°. **4.** 600 ft. **5.** A rhombus, 4.1 in.
6. 45 ft., 53°. **9.** 72°, 72°, 36°. **12.** 34.1 mi., 85°.
13. 248,376 sq. ft. **14.** 2.56 in., 2.7 in. **16.** 450 ft.
18. 10.6 mi. **19.** 59.6 ft. **20.** 2.86 mi.
21. 309 mi., 44°. **22.** 64 ft. **23.** 136 ft.
25. 4.5 in., 5.42 in., 6.25 in. **26.** 58°. **27.** 47.7 ft.
28. 4 mi. **29.** 6.928 sq. in., 1.732 sq. in.
30. 102 mi. **31.** 134,349 sq. yds. **32.** 4.36 mi.
33. 498 yds., 158,535 sq. yds. **35.** 19.6 yds. **36.** 568 yds.
37. 18.9 ft., 17.2 ft. **38.** 179 ft. **39.** 3.34 mi.

PART II

Exercise XV

2. .77 in., 72°. **3.** .432 sq. in., 2.16 sq. in.
4. $AH = 1.12$ in., $HC = .692$ in., $DH = 1.12$ in., $HB = .692$ in.
5. Each distance = .95 in. **6.** .77 in.
7. $AC = AD = BE = BD = CE = 1.81$ in.

Exercise XVI

2. 1 in. **3.** .866 in. **4.** .433 sq. in., 2.598 sq. in.

Exercise XVII

3. 540°, $EAB = 106°$, $AED = 100°$, $EDC =$ [illegible]0°.
4. $BC = 1.47$ in., $CD = 1.12$ in., $DE = 1.02$ in., $EA = .89$ in.
5. $DE = 82$ in., $EA = 1.71$ in.
8. $AB = 2.04$ in., $BC = 3$ in., $CD = 2.95$ in., $DE = 2.4$ in., $EF = 2.92$ in., $FA = 2.69$ in.
9. $BC = 159$ yds., $CD = 195$ yds., $DE = 98$ yds. **10.** 46,428 sq. yds.

Exercise XVIII

1. 64.8 ft. **2.** 73.6 ft. **3.** 115 ft.
4. Course = S 65° W, distance = 44.9 mi.
5. 88.4 yds. **6.** 220 ft., 52.6 ft.
7. Area = 515 sq. ch., $BC = 13.1$ ch., $CD = 17.1$ ch., $DE = 19.8$ ch.
8. 243 yds. **9.** 152,000 sq. yds. **10.** 109 ft., 436 ft.

Exercise XIX

7. 2.2 in., 4 ft. $4\frac{1}{2}$ in., 2 mi. 7 fur., 22.2 yds., 20.3 m. 8 yds. 2 ft. 6 in. respectively.
10. 60 miles to an inch.

Exercise XX

1. $\frac{5}{48}$. **2.** $\frac{7}{576}$. **3.** $\frac{1}{84480}$. **7.** 40°, 60°, 80°. **9.** $\frac{1}{200}$.

Exercise XXII

1. 9. **2.** $52\frac{1}{2}°$, 10 : 30. **7.** $4\frac{1}{2}$.

Exercise XXIII

2. 1.46″, 1.58″, 1.41″, 1.46″.

Miscellaneous Exercises

1. 65.5 ft., 90°, 80°, 76°, 114°, 1965 sq. ft.
2. .74 dm., .66 dm., 88°, 84°, 91°, 97°.
3. $AB = 110$ in., $AC = 87$ in., $DA = 48$ in., $DB = 77$ in., $DC = 56$ in
4. 38.9 ft.
5. $AB = 39.7$ ft., $AE = 80.4$ ft., $AD = 81.9$ ft., $AC = 36.5$ ft.
6. $AE = 423$ ft., $AD = 619$ ft., $AC = 598$ ft., $A = 122°$, $E = 8$
area = 259,805 sq. ft.
7. 24 ft., 384 sq. ft. **8.** 75 ft. **9.** 27.5 ft., 2750 sq.
10. 285 ft., 110°, 65°. **11.** 282
12. $A = 101°$, $B = 112°$, $C = 130°$, $D = 80°$, $E = 159°$, $F = 137°$.
13. 36 ft. **14.** Distance = 80.3 mi., course = N 22°
15. $AB = 411$ in., $BC = 491$ in., $CA = 454$ in.
16. $BC = 18.2$ in., $BD = 39.3$ in., $CD = 23.9$ in., area = 616 sq. in.
17. $AC = 461$ ft., $BD = 552$ ft. **18.** 173
19. 64, 84, 96, 104, and 100 ft. respectively.
20. $AB = 37.9$ ft., $BC = 28.8$ ft., $CD = 25.8$ ft., $DE = 42.4$ ft., $EA = 55.6$
21. 451 ft. **22.** $BC = 246$, $CE = 190$, $BE = 260$, area = 22,2
23. 36 ft. **24.** 428 in., 6
25. Area = 19,800 sq. yds., distance = 534 yds.
26. $A = 138°$, $E = 90°$, $D = 118°$, area = 1907. **27.** 119 ft., 142
28. 53.1 yds. **29.** 22.6 mi. **30.** 111 ft. **31.** 245
32. 32 km., 9.9 km. 26 km. **33.** 137 ft., 237 ft. **34.** 34.2
35. 99,018 sq. yds. **36.** 10 mi. **37.** 15.48 sq. in., 3.87 sq.
38. Heights = 11 and 41 ft. respectively, distance = 30 ft.
39. 18°, 400 yds.
40. $AB = 5.38$ mi., $BC = 9.43$ mi., $AC = 13.3$ mi. Direction is S 13°

= 56 in.

.

. $E = 83°$,

2750 sq. ft.

11. 282 ft.

°.

= N 22° E.

sq. in.

18. 173 ft.

$A = 55.6$ ft.

a = 22,287.

28 in., 67°.

ft., 142 ft.

1. 245 mi.

4. 34.2 ft.

.87 sq. in.

s S 13° W.